Psychic Moving Costs and Mortgage Default with Positive Equity *

David Low†

Consumer Financial Protection Bureau

September 9, 2021

Abstract

Many struggling mortgage borrowers who have home equity lose it through foreclosure. To explain why they do not just sell their homes instead, this paper develops a new model of mortgage default in which homeowners face psychic moving costs. A transparent calibration procedure yields psychic moving costs that are empirically accurate, heterogeneous, and large. The model explains abovewater default: after a liquidity shock, abovewater homeowners with high psychic moving costs default rather than sell in an ex-ante optimal gamble to keep their homes. Psychic moving costs also mostly explain why underwater borrowers so rarely walk away from their homes, another major puzzle in the literature. Relative to a nested model without abovewater default, the full model produces starkly different results in policy experiments. Wealth maximization motivates many fewer defaults, so suing defaulters prevents less than one-fifth as many foreclosures after a drop in house prices. But liquidity constraints alone drive many more defaults, so forbearance prevents between three and seven times more foreclosures after a drop in aggregate income.

*This paper was previously circulated under the title "Mortgage Default with Positive Equity". I thank Brian Bucks, Tom Conkling, Justin Contat, Morris Davis, Scott Fulford, Nicholas Li, Éva Nagypál, and Ryan Sandler for helpful comments that greatly improved this paper. I thank Gizem Koşar, Tyler Ransom, and Wilbert van der Klaauw for data on homeowner wealth by "rootedness" status, and Andreas Fuster, Benedict Guttman-Kenney, and Andrew Haughwout for data on the distribution of household leverage over time. All errors are my own. The views expressed are mine and do not necessarily reflect those of the Consumer Financial Protection Bureau or the United States.

†Email address: david.low@cfpb.gov

1 Introduction

Foreclosures can devastate individual households and the broader economy, with far-reaching and long-lasting consequences. In the 1930s, the social and economic damage wrought by soaring foreclosure rates (a major theme of John Steinbeck's *The Grapes of Wrath*) deepened and prolonged the Great Depression (Bernanke, 1983) and prompted the creation of Fannie Mae, which to this day plays a dominant role in the U.S. mortgage market. More recently, a surge in foreclosures helped fuel the collapse of the housing and financial markets in the late 2000s, leading to what was until COVID-19 the most severe recession in living memory. Informed by this history and facing an unprecedented spike in unemployment at the beginning of the COVID-19 crisis, policymakers and many mortgage servicers moved proactively to prevent a new wave of foreclosures by allowing borrowers to enter "forbearance" and pause mortgage payments without penalty. Over nine million mortgages together worth over $2 trillion entered forbearance between 2020 and 2021 (Haughwout et al., 2021; Cherry et al., 2021), creating a massive disruption in the market that is already the subject of a substantial literature (Capponi et al., 2021; An et al., 2021a; Farrell et al., 2021). Even during economic expansions, hundreds of thousands of homeowners experience foreclosure every year, leading to adverse health outcomes (Currie and Turkin, 2015) and financial distress (Diamond et al., 2020) while exacerbating socioeconomic inequality (Kermani and Wong, 2021).

The dramatic consequences of mortgage default have motivated a large and vibrant literature, yet a foundational puzzle remains unsolved. In existing models a borrower who cannot pay her mortgage faces a choice: she can lose her equity through foreclosure or she can keep her equity by selling her home. The solution is simple: abovewater homeowners sell and do not default. But empirically *most* defaulters have positive home equity,[1] and despite decades of research there is still no model that can explain why so many abovewater homeowners default on their mortgages.

This paper develops a new quantitative model of mortgage default in which non-financial "psychic" moving costs play a key role in homeowners' mobility and default decisions. I develop a transparent and easily-replicated calibration strategy that yields psychic moving costs consistent with empirical estimates, making important contributions to the mobility and housing literatures. After a liquidity shock abovewater homeowners with high psychic moving costs often do not sell their homes and default instead in an ex-ante optimal gamble to avoid moving. The model is the first to generate realistic abovewater default rates, leading to many significant contributions to the mortgage default literature as detailed below.

In the model, agents make housing, consumption, and savings decisions over the lifecycle. Renters allocate expenditures between rent, consumption, and liquid assets. Nonhomeowners may buy homes with long-term mortgage debt. Mortgages are priced endogenously but are subject to exogenous

[1] See Appendix A.1.

Loan-To-Value ("LTV") and Payment-to-Income ("PTI") limits. Agents are subject to income shocks including a discrete and persistent "disaster" shock, and homeowners are subject to house price shocks that can drive them underwater. A homeowner who is current on her mortgage may sell her home, refinance her mortgage, pay it as scheduled, or become delinquent on it.

Mortgage delinquency may or may not lead to foreclosure. A delinquent homeowner can avoid foreclosure by making up her missed payment if her income has recovered. She can also try to avoid foreclosure by selling her home, but because of search frictions in the housing market she might not be able to find a buyer in the little time remaining before foreclosure.[2] Thus an abovewater homeowner who is current on her mortgage but cannot pay it faces a tradeoff. Selling her home immediately allows her to keep her equity but forces her to move. Delinquency may let her avoid moving if her income recovers, but may instead cost her her equity if her income does not recover and foreclosure occurs. The less willing an abovewater homeowner is to move out of her home, the more likely she is to take this gamble and choose delinquency over selling.

Therefore the model must accurately account for homeowners' willingness to move. As is standard the model allows for financial moving costs, but homeowners may also have many non-financial reasons to avoid moving, such as sentimental attachment to a home, a high idiosyncratic match quality with a home, the stress and hassle of a move, a preference for the status quo, etc. Thus the model allows for persistent, heterogeneous, and stochastic moving costs that affect utility but not consumption, i.e. they are "psychic." The shock process for psychic moving costs is calibrated so that, as in the data, recent home buyers are likely to move but longtime homeowners are not. This is precisely the opposite of what the standard consumption-savings framework at the heart of the model would predict without psychic moving costs; this mismatch between the base theory and the data provides clean and transparent parameter identification. Calibrated psychic moving costs are sometimes low and even negative, so that recent homebuyers often move out of homes they just optimally bought. But psychic moving costs must also be quite large on average, so that longtime homeowners move very rarely even though their homes (which were purchased a long time ago) now provide very suboptimal levels of housing consumption. In the calibrated model, homeowners' psychic moving costs are on average 56% of future consumption. For comparison, financial moving costs – which have received far more attention in the literature – are roughly 3% of future consumption, i.e. an order of magnitude smaller.

These results are an important contribution to the broader mobility literature. It is already

[2] Search frictions are important because they imply that homeowners who have equity when they become delinquent face a significant probability of foreclosure; it is much less important that homeowners actually have equity at the time of foreclosure. See Appendix A.2 for an interpretation of the model in which negative equity shocks induced by delinquency (property depreciation, foreclosure fees, etc.), rather than search frictions, prevent homeowners who had positive equity when they became delinquent from selling to avoid foreclosure.

well-known that many households are willing to make large financial sacrifices to avoid moving (e.g. Venti and Wise, 1984; Bartik et al., 1992; Kennan and Walker, 2011; Bayer et al., 2016; Bartik, 2018). The usual interpretation of this result is that people are generally unwilling to move to different states or regions. However Koşar et al. (2021), in a unique survey, elicit choice probabilities in hypothetical scenarios to decompose regional migration costs into different components; they find that psychic costs to move *anywhere* – even to an identical house in the same neighborhood – are quite heterogeneous and on average strikingly large.[3] This paper uses a very different statistical tool (a structural model) and very different data (homeowner moving rates) but obtains moving cost estimates that are surprisingly similar, providing independent validation of the important findings in Koşar et al. (2021).

The calibrated model also represents an important contribution to the very large housing literature (for some reviews, see Davis and van Nieuwerburgh, 2015; Han and Strange, 2015; Piazzesi and Schneider, 2016). While some models have related elements, such as matching frictions in the housing market or exogenous moving shocks, to the best of my knowledge this is the first housing model with heterogeneity in psychic moving costs that is disciplined by data on moving decisions.[4] A potentially serious concern with modeling psychic moving costs is that they are typically unobserved and so could be very difficult to discipline empirically. This paper directly addresses this concern by providing a clean and transparent strategy to identify them; importantly, this strategy could easily be replicated by other researchers interested in any related topic. Psychic moving costs are so heterogeneous and so large on average that they are almost certainly important in many contexts besides mortgage default,[5] so this paper's approach to modeling and calibrating psychic moving costs should be broadly valuable.

This paper makes many contributions to the literature on mortgage default. Because in the model delinquency might not lead to foreclosure and psychic moving costs are often high, after liquidity shocks abovewater homeowners default at realistic rates in a gamble to avoid moving. Other models, by contrast, generate little or no abovewater default and instead rely on negative equity to explain why homeowners default rather than sell.[6] In order to highlight the benefits of modeling abovewater

[3]For example, they estimate that average psychic costs to move to an identical home less than a ten-minute walk away are 59% of permanent household income, excluding the 12% of their sample with moving costs so high they cannot be estimated. But they also estimate that psychic moving costs are zero or negative for 20% of their sample.

[4]Mnasri (2015), Oswald (2019) and Giannone et al. (2020) develop housing models with heterogeneity in the psychic costs of moving across regions. Hembre (2018) instead allows for heterogeneity in psychic default costs, but he explicitly argues these are likely moving costs.

[5]As one illustrative example, the model generates empirically accurate bequests with essentially no bequest motive, since many homeowners optimally avoid paying psychic moving costs by keeping their homes until they die. This novel mechanism may help explain some puzzles in the bequest literature, such as that households without children leave similar bequests to those with children, parents typically do not transfer bequests to their children until death, and homeowners leave much larger bequests than renters (Barczyk et al., 2019).

[6]Section 2 provides a brief overview of the literature.

default, I contrast the main version of the model (hereafter, "$\mathcal{AWD}$" for "abovewater default") with a nested version of the model (hereafter, "$\mathcal{UWD}$", for "underwater default") without search frictions in the housing marking. In $\mathcal{UWD}$ as in other models, abovewater homeowners sell to escape foreclosure and so negative equity is almost a necessary condition for default.[7]

A major benefit of accounting for abovewater default is immediately clear during calibration. In a famous paper, Foote et al. (2008) document a very low underwater default rate; just 6.4% of underwater homeowners in their sample experience foreclosure in three years.[8] I set psychic underwater default costs in $\mathcal{AWD}$ to match this moment. Because $\mathcal{AWD}$ generates realistic abovewater default rates, matching underwater default rates naturally leads it to also match the aggregate default rate. Matching the abovewater, underwater and aggregate default rates prepares $\mathcal{AWD}$ well for policy experiments later in the paper.

In contrast, $\mathcal{UWD}$ does not generate abovewater default, so it must either (1) overstate the underwater default rate to match the aggregate default rate or (2) match the underwater default rate and so understate the aggregate default rate. Facing this unappealing choice for $\mathcal{UWD}$, I follow a standard strategy of setting psychic underwater default costs in order to match the aggregate default rate.[9] This allows $\mathcal{UWD}$ to be used to study the causes and policy implications of default in Section 6, but it requires $\mathcal{UWD}$ to overstate the underwater default rate by a factor of nearly three.

The surprisingly low default rate of underwater homeowners is a major and enduring puzzle; Foote and Willen (2018) call it one of two "central" questions in the literature. Because $\mathcal{AWD}$ matches underwater default rates, it can be used to study them. For abovewater homeowners, high psychic moving costs discourage selling and lead to default. For underwater homeowners, selling is not a viable option anyway, and the only choice is between paying the mortgage or defaulting. Defaulting leads to foreclosure and therefore moving, so high psychic moving costs make underwater default less likely. $\mathcal{AWD}$ needs psychic underwater default costs that are only about 13% of average psychic moving costs for underwater homeowners, so in this sense psychic moving costs explain about 89% of the underwater default rate puzzle.

Next I use the two versions of the model to study the relationship between equity and default, a primary focus of the literature for decades. Foote et al. (2008) estimate that homeowners with a

[7]Like many other quantitative models, $\mathcal{UWD}$ does generate limited default by homeowners who have little equity and are technically abovewater but are effectively underwater, after accounting for financial moving costs.

[8]Other papers that document surprisingly low underwater default rates include Foster and Order (1984), Vandell (1995), Foote et al. (2008), Deng and Quigley (2013), Bhutta et al. (2017), and Gerardi et al. (2018).

[9] Other models that target the aggregate default rate include Demyanyk et al. (2017), Schelkle (2018), Krivenko (2019), Hu (2019), Kaplan et al. (2020), Campbell et al. (2020), and Guren et al. (2020). A few models match the underwater default rate instead (Laufer, 2018; Hembre, 2018; Ganong and Noel, 2021). These different calibration strategies explain why models in the former category set default costs to be low (so that underwater default is common enough to compensate for the lack of abovewater default), while models in the latter category set psychic default costs to be quite high (around 25-70% of *lifetime* consumption so that underwater default is as rare as it is in the data.)

measured LTV of 120 are roughly five times more likely to experience foreclosure than those with a measured LTV of 80.[10] In $\mathcal{UWD}$ they are about 80 times more likely, because $\mathcal{UWD}$ generates little abovewater default and exaggerates the underwater default rate to compensate.[11] $\mathcal{AWD}$ generates a much more accurate – though still slightly high – ratio of eight-to-one. As shown below, $\mathcal{AWD}$'s superior ability to match the relationship between equity and default has important implications in policy experiments.

Moreover, these differences in the role of negative equity induce large differences in the role of adverse shocks. Underwater homeowners with the disaster income shock are very likely to default regardless of parameter values in both $\mathcal{AWD}$ and $\mathcal{UWD}$. Thus both models effectively set the rate of underwater default with no disaster shock ("strategic default") to match their calibration targets. $\mathcal{AWD}$ targets the low underwater default rate, which requires that just 7.6% of defaults be strategic. But in $\mathcal{UWD}$ 53.3% of defaults are strategic, because in order to match aggregate default rates $\mathcal{UWD}$ requires more underwater default than the disaster income shock can generate. Ganong and Noel (2021) and Low (2021) estimate that only around 4% of defaults are strategic.[12] Thus accounting for abovewater default greatly improves $\mathcal{AWD}$'s ability to match extremely low strategic default rates. This also has important implications in policy experiments.

Models of mortgage default are frequently used to understand the effect on foreclosure rates of events or policies that affect equity, such as the drop in house prices during the Great Recession. $\mathcal{AWD}$'s unique ability to match the relationship between negative equity and default risk provides it with a major advantage in this context and leads to large differences with $\mathcal{UWD}$. For example, after an unanticipated, one-time drop in house prices by .10 log points (about 9.5%), foreclosures increase by 49.3% in $\mathcal{AWD}$ and by a much larger 234.9% in $\mathcal{UWD}$. Thus accounting for abovewater default is especially important in policy experiments when negative equity becomes common, as it was during the Great Recession and may be again.

Finally, abovewater default has important policy implications for the recession induced by the COVID-19 pandemic. In 2020 tens of millions of Americans became unemployed yet house prices grew by 8.4% and negative equity was virtually nonexistent.[13] After an unexpected, one-time

[10] Estimates from other papers are similar. For example, Elul et al. (2010) and Fuster and Willen (2017) both find that homeowners with a combined LTV ("CLTV") above 120 are less than four times more likely to default than those with a CLTV between 70 and 80. Fuster et al. (2018) find that delinquency rates for properties with CLTVs between 60-80 are roughly one-tenth of those with CLTVs above 120. Laufer (2018) finds that default rates for homeowners with LTVs between 75 and 100 are a quarter of those with LTVs between 100 and 125. For more information on how the relationship between equity and default has changed over time, see An et al. (2021b).

[11] Predictions from the models account for the fact that Foote et al. (2008) use data from a recourse state and for measurement error. Measurement error is the only reason any default is measured to occur at an LTV of 80 in $\mathcal{UWD}$.

[12] The findings of Ganong and Noel (2021) and Low (2021) contrast with those from Gerardi et al. (2018), who find that 38% of defaults are strategic. As both Ganong and Noel (2021) and Low (2021) demonstrate, the discrepancy arises because Gerardi et al. (2018) label defaults without observed income shocks as strategic when, in reality, they occur because of adverse liquidity shocks that are not observed in their data.

[13] See https://www.zillow.com/research/zillow-december-2020-market-report-28684/ and https:

increase in the probability of a disaster shock, with no change in house prices, the foreclosure rate increases by 79.5% in $\mathcal{AWD}$ and by 43.1% in $\mathcal{UWD}$. The increase in foreclosures is nearly twice as large in $\mathcal{AWD}$ because in $\mathcal{AWD}$ even abovewater homeowners are at risk of defaulting with the disaster shock. Introducing a stylized version of forbearance prevents between 3.0 and 7.0 times more foreclosures in $\mathcal{AWD}$ than $\mathcal{UWD}$. Forbearance is more effective in $\mathcal{AWD}$ because it helps abovewater homeowners avoid foreclosure; in $\mathcal{UWD}$ abovewater homeowners do not need help avoiding foreclosure, even after the disaster shock, because they can sell their homes with probability one. This finding suggests that forbearance may have been much more effective in preventing foreclosures during the 2020 recession than standard models would predict.

The rest of this paper is organized as follows. Section 2 places this paper within the mortgage default literature. Section 3 presents the base model used to develop both $\mathcal{AWD}$ and $\mathcal{UWD}$. Section 4 parameterizes the two versions of the model. Section 5 discusses the fit of the models to the data, while Section 6 explores the causes of default and the results of policy-relevant experiments in the two models. Section 7 summarizes and suggests some directions for future research.

2 Literature Review

There is a very large literature on mortgage default; see Vandell (1995) for an early review and Foote and Willen (2018) for a more recent one. In existing models, defaulters are classified as either "strategic" or "double-trigger." A strategic defaulter can afford to pay her mortgage but "strategically" chooses to walk away from it instead because she is underwater. A double-trigger defaulter cannot afford to pay her mortgage because of one trigger (e.g. unemployment) and does not sell her home because another trigger (e.g. a drop in house prices) has driven her underwater. Clearly, existing models rely heavily on negative equity to explain default.[14] However, many quantitative models do generate limited default by homeowners who are technically abovewater but are "effectively" underwater after accounting for financial selling costs. $\mathcal{UWD}$ does as well, so comparing $\mathcal{UWD}$ with $\mathcal{AWD}$ demonstrates the value of accounting for abovewater default rates beyond what financial selling costs can explain.

The starting point of $\mathcal{AWD}$ is a lifecycle consumption-savings model with housing and mortgages that owes a great deal to previous work (e.g. Jeske et al., 2013; Chatterjee and Eyingungor, 2015; Corbae and Quintin, 2015; Campbell and Cocco, 2015; Schelkle, 2018; Laufer, 2018). Within an otherwise standard framework, the model adds three ingredients that are unusual but not entirely

//libertystreeteconomics.newyorkfed.org/2021/09/if-prices-fall-mortgage-foreclosures-will-rise/.

[14] Riddiough (1991) develops an early and extremely rare exception. In one of his models, negative equity plays no role because a liquidity shock is both necessary and sufficient for default.

new. The combination of the ingredients is novel and allows it to generate realistic abovewater default.

First, in nearly all models of mortgage default, mortgage nonpayment leads directly to foreclosure. However Herkenhoff and Ohanian (2019) document that delinquency is in fact an extended process with an uncertain outcome. Motivated by this evidence, they develop a model in which foreclosure takes more than one period so homeowners who become delinquent can cure their delinquency to escape foreclosure. However, all homeowners are underwater in their calibrated model. This paper follows Herkenhoff and Ohanian (2019) in allowing foreclosure to take more than one period; it extends their work by studying the decisions of abovewater homeowners in this kind of scenario.

Second, I assume that a delinquent homeowner has little time remaining before foreclosure and so might not be able to find a buyer for her home; thus the probability of home sale after delinquency is less than one. Other models implicitly set the probability of home sale after delinquency to zero (since delinquency always leads to foreclosure), so search frictions are *less* substantial in this paper than others. Still, this paper bears some resemblance to several macroeconomic models with mortgage default that also allow for search frictions (Hedlund, 2016a,b; Garriga and Hedlund, 2019; Head et al., 2020) and so generate some abovewater default. The focus of these papers is not abovewater default but macroeconomic topics such as the interplay between search frictions and the Great Recession. Accordingly, they feature realistic mechanisms missing from this paper (e.g. general equilibrium interactions between the housing and mortgage markets) but do not feature mechanisms included in this paper to provide a strong theoretical foundation for abovewater default. In particular, in these other models delinquency leads to foreclosure. This means these papers do not study why abovewater homeowners do not simply sell their homes early in the delinquency process when time is plentiful and search frictions are minimal, a key question answered in this paper by psychic moving costs.

Third, psychic *moving* costs are extremely rare in the literature but psychic *default* costs are quite common, and psychic moving costs are often offered as one explanation for them. However most models match aggregate default rates without abovewater default; this requires that they generate much more underwater default than in the data, which in turn requires psychic default costs that are much lower than the psychic moving costs in this paper.[15] One very important exception is Hembre (2018); he calibrates heterogeneous psychic default costs to be on average enormous – 70% of lifetime consumption – in order to match underwater default rates, and he specifically argues that these default costs likely represent borrowers' sentimental attachment to their homes. Laufer (2018) allows for both homogenous psychic default costs (calibrated to match underwater default rates)

[15]See footnote 9.

and homogenous psychic moving costs (calibrated to match moving rates). He needs psychic moving costs equal to an extremely large 54% of future consumption to explain why so many homeowners failed to sell early in the Great Recession when it became clear (at least to agents in the model) that house prices would continue to fall. This finding provides an earlier and important, but so far mostly overlooked, indication that homeowners' mobility and default decisions are not explained by financial considerations alone. This paper builds upon Hembre (2018) and Laufer (2018) by developing a new and easily-replicated methodology to calibrate heterogeneity in psychic moving costs to match moving rates. Moreover this paper shows that realistic psychic moving costs explain not only low underwater default rates but also high abovewater default rates, and are therefore fundamental for modeling mortgage default.

To summarize, this paper has three unusual ingredients: (1) borrowers *may* be able to get current after delinquency, (2) borrowers *may* be able to sell after delinquency, and (3) psychic moving costs. Together these generate the novel and crucial mechanism in this paper: abovewater homeowners sometimes default after liquidity shocks in a losing gamble to avoid moving. Figure 1 provides a simple graphical illustration of the main differences between this model and double-trigger models in which liquidity shocks also drive default. The next section embeds this simple framework in a much more realistic (and complicated) quantitative lifecycle consumption-savings model.

3 Model

3.1 Overview

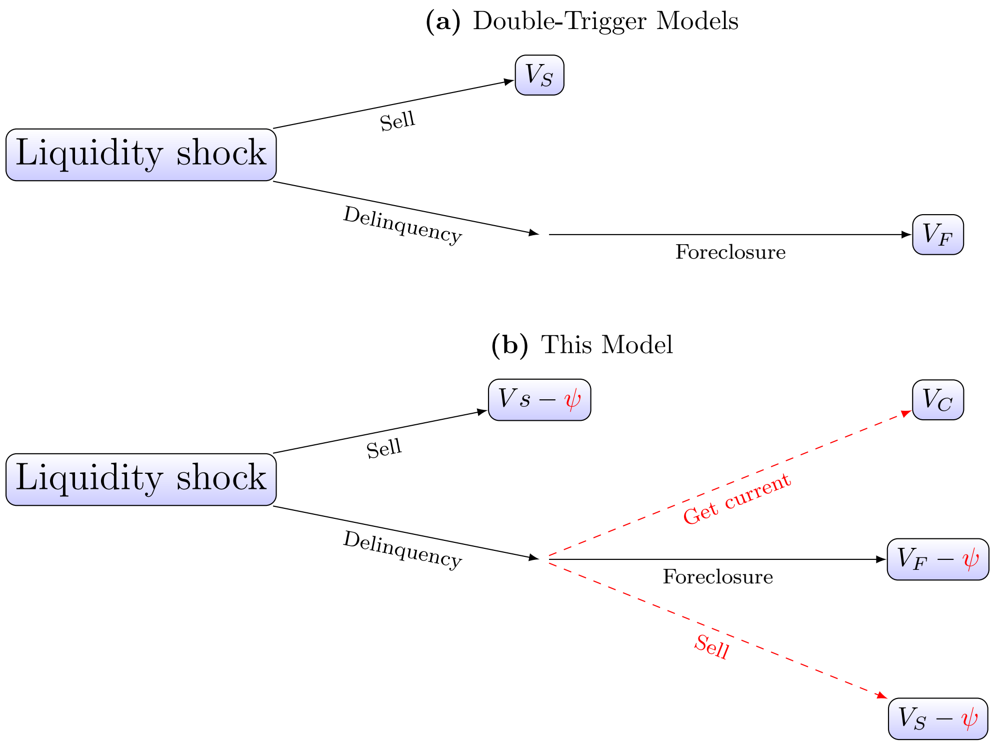
This section builds a quantitative lifecycle model of mortgage default. Value functions are in the Appendix in Section A.3.

Income is exogenous, stochastic before retirement and certain thereafter. Agents can save in liquid, risk-free, one-period bonds, and they choose levels of nonhousing and housing consumption. Agents must either own homes or rent them. Home purchases can be financed with long-term, refinanceable and defaultable mortgage debt, which is priced competitively by financial intermediaries. Mortgage delinquency can lead to foreclosure, which extinguishes the agent's mortgage debt, forces her to leave the home and precludes her from obtaining another mortgage for a period of time.

Because the model distinguishes between delinquency and foreclosure, throughout this paper I use the term "default" to mean a delinquency that leads to foreclosure.[16]

[16] In the mortgage market, "default" is a vague term that can mean anything from 60-day delinquency to foreclosure. The definition that I use is comparable to that used in most other mortgage default models, in which delinquency leads to foreclosure.

Figure 1: Comparison with Double-Trigger Models



NOTE: Figure shows the decisions and associated value functions facing a mortgage borrower that, because of a liquidity shock, cannot afford to pay her mortgage this period. Top panel shows a standard double-trigger model in which delinquency leads to foreclosure so abovewater homeowners sell and do not default. Bottom panel shows the basic mechanism in this paper: delinquency may or may not lead to foreclosure so abovewater homeowners sometimes default. ψ indicates psychic moving costs. Dashed lines indicate options the borrower may or may not have. Differences between the two models are in red.

3.2 Environment

Demographics Time is discrete. A period is a year. Agents face an age-dependent chance of death every period, retire in period T^{retire}, and die with certainty in period T.

Income Consumers receive an exogenous, stochastic income flow $\{y_t\}$. Income follows a deterministic trend g_t and is subject to lognormal transitory and permanent shocks, as well as a binary, persistent "disaster" shock. Specifically,

$$\log(y_t) = g_t + z_t + \delta_t + \epsilon_t \tag{1}$$

where z_t, δ_t, and ϵ_t are the stochastic permanent, persistent, and temporary components of income, respectively.

The permanent component z_t follows the random walk

$$z_t = z_{t-1} + \eta_t$$

The shocks to temporary income (ϵ_t) and to permanent income (η_t) have mean zero and are normally distributed.

The persistent shock δ_t follows a two-state Markov process. This disaster shock represents a significant drop in income (or increase in expenses) and drives many defaults in the model. This process is calibrated to match the data as described in Section 4.

After retirement, the variance of the temporary and permanent shocks is set to zero, as is the probability of entering (or staying in) the disaster income state.

It is useful to separately define the portion of current income that is predictive of future income.

$$\log(\overline{y}_t) = g_t + z_t + \delta_t \tag{2}$$

$\overline{y_t}$ is the income concept used to underwrite loans.

Liquid Assets Agents can save or borrow in a liquid, risk-free asset a, with rate of return R. Agents' borrowing limit depends on $\overline{y}_t$, with

$$a_{t+1} \geq \xi \overline{y}_t \tag{3}$$

Preferences Consumers discount the future at rate β. Households value both consumption c and housing services, which depend on the market-priced quantity h of housing consumed.

Following most of the literature, I assume that utility u is CES between c and h, and CRRA over time. Let θ denote the elasticity of substitution between c and h, let ω denote the weight on c, and let γ denote the coefficient of relative risk aversion. In the utility function, c and h are deflated by effective family size e_t, which is a deterministic function of age.

Together, these assumptions give the per-period utility function:

$$u_t(c, h) = \frac{[(\omega \frac{c}{e_t}^{\frac{\theta-1}{\theta}} + (1-\omega)\frac{h}{e_t}^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}}]^{1-\gamma}}{1-\gamma} \tag{4}$$

Agents value wealth at death according to a standard warm-glow bequest function B taken from De Nardi and Yang (2014). Let w denote an agent's net wealth, and let $c^*(w)$ and $r^*(w)$ denote optimal consumption and rent, respectively, from the one-period renter's problem with cash-on-hand w. Then $B(w)$ satisfies:

$$B_t(w) = \upsilon_1 u_t(c^*(w + \upsilon_2), r^*(w + \upsilon_2), m) \quad (5)$$

υ_1 controls the strength of the bequest motive, while υ_2 determines the extent to which bequests are a luxury good.

Housing Households must either own a home or rent one. Renters can spend an arbitrary amount on rent r_t, and receive housing services $h_t = r_t$.

Home buyers choose a price p of home to buy. A home with price p provides housing services $h_t = \kappa p_t$.

After an agent buys a home, p is subject to permanent lognormal shocks, with:

$$\log(p_t) = \log(p_{t-1}) + \epsilon_t^p$$

where ϵ_t^p has mean zero. These house price shocks are important because they can drive homeowners underwater.

Home buyers pay proportional buying costs $\phi_b p$. To maintain a home, owners must pay proportional property taxes $\zeta_t p_t$ and maintenance fees $\zeta_m p_t$.

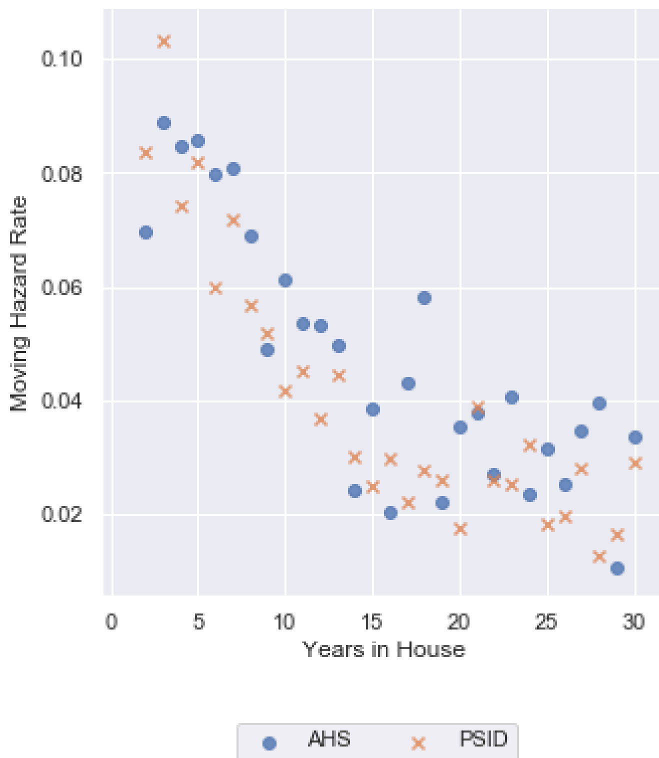
In order to sell a home, agents must find a buyer and then pay a proportional selling cost $\phi_s p_t$. Interested sellers who are current on their mortgage find a buyer with probability one.[17] The sales process for homeowners in foreclosure, who have less time to find a buyer, is described later.

Psychic Moving Costs The modeling ingredients for housing described above are common in lifecycle consumption-savings models. In these kinds of models, a home is typically only valued as a source of housing services h_t. Bigger homes provide more housing services but also incur more costs, so a homeowner's valuation of h_t depends on how it compares to the level of housing services she wants to pay for. Soon after a home is purchased, h_t is likely close to optimal, because otherwise the owner would have chosen a different h_t. As time progresses, lifecycle considerations and income and house value shocks drive h_t further and further away from the level of housing consumption desired by the homeowner. Financial transaction costs ϕ_s and ϕ_b create an inaction region inside of which the financial costs of moving exceed the benefit of selecting a new h_t, and the agent does not move. The probability that an agent moves is essentially the probability that enough shocks have accumulated to move the agent out of this inaction region. Therefore, according to these kinds of models, homeowner mobility rates should start out low and increase with time.

[17] This assumption is somewhat unrealistic. I make it primarily so that all foreclosures in the model are fully endogenous, in the sense that before foreclosure occurs a homeowner must affirmatively choose delinquency over selling the home.

Figure 2 plots empirical mobility rates as a function of time in the home. The pattern is nearly the opposite of that predicted by the standard consumption-savings framework. Since this framework makes counterfactual predictions about homeowners' probability of moving, it almost certainly also makes counterfactual predictions about homeowners' willingness to move, which is a key force in the model.

Figure 2: Moving Hazard Rates as a Function of Housing Tenure



Notes: Data is from the American Housing Survey (AHS) and the Panel Survey of Income Dynamics (PSID).

In order to accurately model homeowners' willingness to move, I therefore allow for heterogeneity in psychic moving costs. As detailed in Section 4, these costs are disciplined in part by using the data in Figure 2. Several other papers (e.g. Kennan and Walker (2011), Bayer et al. (2016), Bartik (2018), Ngai and Sheedy (2019), Oswald (2019), and Giannone et al. (2020)) also allow for psychic moving costs that are calibrated to match moving rates. Importantly, however, this is the first paper to do so in a consumption-savings model with psychic costs to move *anywhere* (and not just to different states or regions.) This means that the modeling and calibration choices I make provide a new and valuable framework for modeling psychic moving costs in a major class of models.

Homeowners pay psychic moving costs ψ_t whenever they move, whether through a sale or foreclosure. ψ_t multiplies both current and future utility if an agent moves. ψ_t is heterogeneous across agents, persistent over time, and stochastic. It is an important state variable that drives many results in this paper.

Immediately after buying a home, an agent's new ψ_t is drawn from a Uniform distribution Ψ_t^I with minimum 0 and maximum $\overline{\Psi}_t^I$. Note that $\overline{\Psi}_t^I$ varies by age t; in particular, older home buyers may on average have higher initial ψ than younger ones. Every period after the purchase of a home, ψ_t is subject to stochastic shocks drawn from a Cauchy distribution Ψ^S. Ψ^S has location parameter Ψ_l^S and scale parameter Ψ_s^S, which are time-invariant.

Because during calibration the model targets when homeowners move out of a home, it is important that the model match when they move in, since lifecycle considerations play an important role in housing decisions. Therefore I assume there is a fraction Π_{NB} of agents who at $t = 0$ are exogenously prohibited from buying a home. Agents who are prohibited from buying homes have a constant probability π_{nb} of a life event (e.g. marriage or birth of a child) that permanently removes the restriction. Agents who cannot buy homes cannot get mortgages, so they play almost no role in the results. The point of Π_{NB} and π_{nb} is to give the model the degrees of freedom to match the homeownership profile closely.

Mortgages Mortgages are fixed-rate, fully-amoritizing loans with interest rate R_m that last until period T. Thus a mortgage at time t is fully characterized by its scheduled payment m.

When buying a home or refinancing, a borrower agrees to pay m to a lender every period in exchange for a loan. The size of the loan L is determined in a perfectly competitive market. However, at origination, mortgages are subject to an exogenous LTV limit $\text{LTV}^{\max}$ and an exogenous "Payment-to-Income" ratio (PTI) limit.

The PTI limit is:[18]

$$\frac{m_t + \zeta_t p}{\overline{y_t}} \leq \text{PTI}^{\max} \tag{6}$$

After origination, the outstanding balance $\Pi(m,t)$ on a mortgage is the cost of buying back the nominal sequence of payments, $\{m_t, m_{t+1}, ..., m_T\}$ at the interest rate R_m:

$$\Pi(m,t) = m\frac{(1-(R_m^{-1})^{T-t-1})}{(1-R_m^{-1})} \tag{7}$$

This in turn defines the LTV of a homeowner who is current on her mortgage:

$$\text{LTV}(p,m,t) = 100 * \frac{\Pi(m,t)}{p} \tag{8}$$

[18]Note the PTI limit depends on the mortgage payment, taxes, and income, but not other debt the consumer may have. Therefore it is a "front-end" limit and not a "back-end" one.

Sellers must repay their mortgage at the time of sale.

Mortgages do not have prepayment penalties and so agents can pay down their mortgage balance at cost $\Pi(m_{old}, t) - \Pi(m_{new}, t)$ if $m_{new} < m_{old}$. However, to extract equity and choose $m_{new} > m_{old}$, agents must refinance by paying proportional financial costs $\Phi_R \Pi(m_{new}, t)$..

Consumer Beliefs Empirically, a substantial fraction of homeowners are "hand-to-mouth" (HtM), i.e. have nearly no liquid wealth to smooth consumption (Kaplan and Violante, 2014). After a disaster shock, homeowners with liquid wealth are likely to pay their mortgage with savings, while homeowners without it are likely to default. Therefore it is important for the model to match the fraction of HtM homeowners.

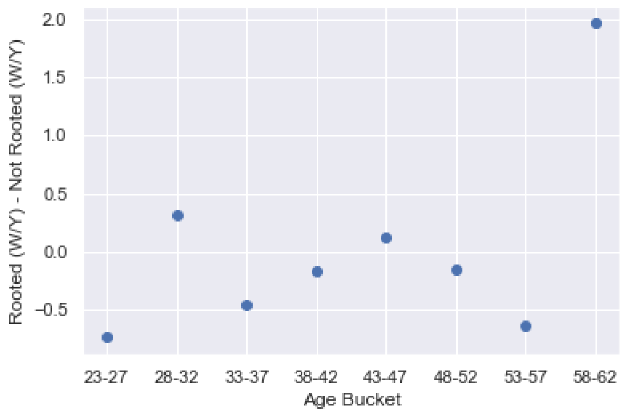
When calibrated to match the data as described in Section 4, this model includes both a substantial chance of a disaster shock and high psychic moving costs. Together these would generate very strong precautionary motives, and hence very few HtM homeowners, if agents had standard preferences and beliefs. Whether (and in what contexts) standard preferences and beliefs actually describe households' savings behavior is the subject of a very large and active literature.[19] A common finding is that households' precautionary savings behavior can deviate significantly from that predicted by the standard consumption-savings framework, but unfortunately there is no consensus on the best way to modify this framework to fit the data. The precise relationship between risk and savings is outside the scope of this paper, but to obtain credible results the model needs to match the fraction of HtM homeowners. Therefore I introduce a simple and plausible mechanism to do so by assuming that agents' perceived probability of moving from the good income state to the disaster income state is $\hat{\Delta_{gb}}$, which is allowed to differ from the true value Δ_{gb}. As described in Section 4, $\hat{\Delta_{gb}}$ is set to match the fraction of homeowners that are HtM. This is the only deviation from rational expectations in the model.

A potential concern with this strategy is that it could lead the model to understate the correlation between ψ_t and precautionary wealth. For example, it is plausible that HtM households could disproportionately have zero or low ψ_t, while households with high ψ_t could have substantial precautionary wealth. If this were the case, abovewater homeowners would be unlikely to default after a liquidity shock; those with low ψ_t would sell their homes, while those with high ψ_t would have the precautionary wealth to smooth their consumption without becoming delinquent. To investigate this concern, Figure 3 plots the difference between the median nonhousing wealth of high ψ_t households and that of low ψ_t households in data provided to me by Koşar et al. (2021).

[19] This literature is far too large to review here, but for just a few examples, see Carroll and Samwick (1997), Fulford (2015), Olafsson and Pagel (2018), Francesco D'Acunto and Weber (2020), Attanasio et al. (2020), and Ganong et al. (2020).

The figure indicates that the correlation between ψ_t and nonhousing wealth is very weak or even negative, and so the model will not understate this correlation.

Figure 3: Differences in Median Nonhousing Wealth Over Income by "Rootedness"



Notes: Figure shows, by age bucket, the median nonhousing wealth normalized by income of "rooted" homeowners minus that of "not rooted" homeowners. Negative values indicate that "not rooted" homeowners have more nonhousing wealth than "rooted" homeowners. "Rootedness" is a strong proxy for psychic moving costs; see Koşar et al. (2021) for more information. I thank the authors for providing me with this data.

Delinquency Homeowners who are current on their mortgage can choose to become delinquent. This allows them to avoid paying their mortgage, property taxes, and property maintenance fees for the period. As in Herkenhoff and Ohanian (2019) but unlike most other models of default, delinquency does not lead to certain foreclosure at the end of the period. Rather, homeowners who become delinquent begin the next period at risk of foreclosure.

Foreclosure A homeowner who chose delinquency in period $t-1$ receives a foreclosure notice at the beginning of period t. At this point, she owes her lender what a current homeowner would owe $\Pi(m,t)$ plus her missed mortgage payment and back taxes with penalty interest $R_m^d(m+\zeta_t p)$. To get current she must also repair the depreciated home at cost $R_\zeta(\zeta_m)p$. Her new LTV is:

$$\text{LTV}^{\text{f}}(p,m,t) = 100 * \frac{\Pi(m,t) + R_m^d(m+\zeta_t p)}{p - R_\zeta \zeta_m p} \tag{9}$$

Unless otherwise noted, the LTV reported in tables and figures accounts for the effects of delinquency, i.e. it is determined by Equation 8 for current homeowners and by Equation 9 for homeowners in foreclosure.

Let $\Phi_F(p,m,t) = R_m^d(m+\zeta_t p) + R_\zeta(\zeta_m)p$ denote the costs the borrower must pay to become current. At this point she still has two chances to avoid foreclosure.

First, she may list her home for sale at price $p^l \leq p$, in which case the property sells with probability $\pi_s(p^l)$.[20] After a sale, the agent must pay $\Pi(m,t)$ and $\Phi_F(p,m,t)$ in full. She keeps any proceeds and becomes a non-homeower without a foreclosure flag.

Second, the agent may become current and keep her home by paying the lender $\Phi_F(p,m,t)$. Agents who are not in the disaster income state are allowed to spend all cash-on-hand, and borrow up to their borrowing limit, to do so. Agents in the disaster income state are prohibited from getting current.[21] If the agent becomes current in this way, her assets are adjusted downwards appropriately, she keeps her home and her mortgage status becomes current. Like other current homeowners, she then chooses whether to make her next mortgage payment on time, sell her home instantly without search frictions, refinance her mortgage, or become delinquent again.

If the property is not sold and the owner does not become current, the lender forecloses on the property and sells it at auction. Net of costs, the lender receives $(1-\chi_D)p$ from the foreclosure. If this exceeds what the lender is owed, the excess is returned to the agent. Otherwise, the agent receives nothing.[22] She becomes a nonhomeowner with a default flag. If the agent was effectively underwater at the time of foreclosure, she also pays psychic foreclosure costs ψ_{F}. As is standard, ψ_{F} is introduced largely to give the model another free parameter to target default rates in calibration. It can be viewed as a proxy for any other factor missing from the model which decreases underwater default rates; there may be moral costs of strategic default (Guiso et al., 2013), underwater borrowers may worry about being sued if they default (Ghent and Kudlyak, 2011), they may not be aware they are underwater (Chan et al., 2016a), they may believe (correctly or incorrectly) that house price drops are more temporary than they are in the model (Foote and Willen, 2018), etc.

4 Parameterization

As is typical, parameterization is a two-step process. First, I set many parameters to values used elsewhere in the literature, or to values that can be estimated without the use of the structural model. Second, I take the parameter values from the first stage as given, and choose the remaining parameters to minimize the distance between selected empirical moments and model output. During

[20] Recall that current homeowners can sell with probability one. This ensures that all delinquencies are fully endogenous, but means that current homeowners cannot face a realistic tradeoff between price and time-to-sale and so they can only sell at price p. Therefore homeowners in foreclosure are restricted to $p^l \leq p$ so that homeowners do not have an incentive to become delinquent in order to sell their homes at a higher price.

[21] In reality, the difficulty of reinstating a mortgage depends on state laws, the precise timing of income and spending, the willingness of the lender to work with the borrower, the other borrowing opportunities still available to a person in foreclosure, and many other factors. Moreover, because current homeowners in the model can sell their homes instantaneously, agents who have equity and can become current will always do so, if only to sell the home instantly after. The model is not high-frequency enough to address these issues endogenously, so instead I assume that the ability to become current is partly exogenous as determined by the income state.

[22] The agent also does not need to pay anything in the baseline model, i.e. there is no lender recourse.

these steps, I parameterize the model so that it is applicable to a "normal" housing market. Often, this means using moments and data from 1998-2001, before the 2007 foreclosure crisis or the housing boom that preceded it. The details of the two steps are described in turn.

I calibrate two versions of the model. I calibrate $\mathcal{AWD}$ by setting the probability a prospective house seller can find a buyer during foreclosure to match empirical estimates of sale probabilities as a function of list price from Guren (2018).[23] This allows the model to generate both abovewater and underwater default and so gives it the degrees of freedom to match both the underwater default rate and the aggregate default rate in the second stage of calibration.

The calibration strategy for $\mathcal{UWD}$ eliminates search frictions from the model by setting the probability of sale during foreclosure to one. Thus $\mathcal{UWD}$ reproduces the standard prediction that effectively abovewater homeowners always sell to avoid foreclosure. This restriction removes the model's ability to match both the underwater and aggregate default rates, and so $\mathcal{UWD}$ must choose which to match.[24] Facing this choice for $\mathcal{UWD}$ I follow most of the literature and target the aggregate default rate in calibration.[25] This choice allows $\mathcal{UWD}$, like $\mathcal{AWD}$, to be used to study the causes and policy implications of default in general. However as will be shown later, this requires $\mathcal{UWD}$ to substantially overstate the underwater default rate in order to compensate for the lack of abovewater default.

4.1 First Stage

The values for parameters that can be estimated without the use of the structural model are described first.

Demographics: Households begin life at age 23, retire at 65, and die with certainty at 85. Age-specific mortality rates are from the 2008 National Longitudinal Mortality Survey (NLMS).

Income: As the empirical counterpart of income in the model, I use real, after-tax, nonasset income at the household level from the PSID.

[23] These estimates are for sale probabilities for all homeowners; I am not aware of estimated sale probabilities as a function of list price for homeowners in foreclosure. For a given list price, a homeowner in foreclosure could have a higher sale probability because of increased effort, or a lower one because of the stigma surrounding properties in foreclosure. As shown later in Table 4, the model reproduces the unconditional probability a homeowner in foreclosure sells her home, suggesting this issue is minor.

[24] This choice can be interpreted as being between two different interpretations of the data. Under one interpretation, the abovewater and underwater default rates measured in the data are correct. Under this interpretation, $\mathcal{UWD}$'s inability to generate abovewater default is a shortcoming. Under the other interpretation, $\mathcal{UWD}$'s prediction of near-zero abovewater default is correct and defaulters measured in the data to be abovewater are actually underwater. Under this second interpretation, underwater default rates are substantially higher than they appear in the data, and so calibrating $\mathcal{UWD}$ to match aggregate default rates is the correct strategy.

[25] For examples of models that target the aggregate default rate, see Jeske et al. (2013), Demyanyk et al. (2017), Schelkle (2018), Krivenko (2019), Hu (2019), Kaplan et al. (2020), Campbell et al. (2020), and Guren et al. (2020). Laufer (2018), Hembre (2018), and Ganong and Noel (2021) target the underwater default rate instead.

The deterministic component of income g_t is the empirical median by age. The retirement replacement ratio is .866, which is the empircal average.

There are five remaining parameters that govern the income process: the variance of temporary shocks σ_ϵ^2, the variance of permanent shocks σ_η^2, the probability of a persistent disaster shock, the probability a disaster shock ends, and the size of a disaster shock.

Values for these five parameters are set to minimize the distance between six moments from simulated and actual data, using fairly standard methodology inspired by Carroll and Samwick (1997). For estimation, I keep data from households with a head between the ages of 23 and 59. Observations with annual income below \$1,000 may have left the labor force and so are dropped to avoid overstating the size or frequency of disaster income shocks. To remove predictable income changes, I regress log income on a rich set of observables.[26] Let $\tilde{y}_t$ denote log residual income, i.e. log income minus predicted log income. Estimated parameter values, their interpretation, and the moments that help to identify them are reported in Table 1.

Table 1: Income Parameter Estimates

Parameter	Interpretation	Value	Moment
σ_ϵ^2	Var. of temporary income shock	.0304	$var(\tilde{y}_t - \tilde{y}_{t-3})$
σ_η^2	Var. of permanent income shock	.0116	$var(\tilde{y}_t - \tilde{y}_{t-7})$
Δ_{gb}	Prob. of a disaster income shock	.0218	$P((\tilde{y}_t - \tilde{y}_{t-1}) < -1)$
Δ_{bg}	Prob. a disaster income shock ends	.5370	$P((\tilde{y}_t - \tilde{y}_{t-1}) > .5)\|(\tilde{y}_{t-1} - \tilde{y}_{t-2}) < -1)$
δ_b	Size of a disaster income shock	1.2536	First percentile of $(\tilde{y}_t - \tilde{y}_{t-1})$

Note: Table displays the estimated income parameters, along with their economic interpretation, estimated value, and empirical moments that help identify them. A sixth moment, $P((\tilde{y}_t - \tilde{y}_{t-5}) < -1)$ is also targeted in estimation, and so the income process is overidentified. The parameter-moment correspondence is for intuition only; formally, every moment helps identify every parameter.

Liquidity shocks besides income shocks, such as expense shocks, health shocks, and divorce are frequent triggers of mortgage default (Ganong and Noel, 2021; Low, 2021), so the model needs to account for them. High-quality data on how precisely these shocks affect households is not available, and explicitly modeling all of them is not feasible, so I account for them indirectly.[27] To keep the model as tractable as possible, I assume that other liquidity shocks have the exact same magnitude and persistence as disaster income shocks. Moreover, disaster income shocks and other disaster liquidity shocks never occur at the same time.[28] This means that disaster liquidity shocks can be accounted for in the model solely by adjusting Δ_{gb} upwards appropriately.

In the American Survey of Mortgage Borrowers (ASMB), Low (2021) finds that virtually all defaults are triggered by a liquidity shock, but less than half of defaulters report a substantial

[26] Details are available upon request.

[27] For an earlier version of this paper that explicitly modeled the role of divorce in default, see Low (2015).

[28] This assumption is not realistic but it is necessary because there is no way to discipline another approach. Thus the model likely understates the intensity of liquidity shocks triggering default.

decrease in income of the kind measured above; this would imply that over half of defaults are triggered by liquidity shocks that are not large income shocks so the value for Δ_{gb} estimated in Table 1 is too low by a factor of more than two. However, this adjustment factor may be too high because it is possible for default to be triggered by small or temporary income shocks. In the ASMB two-thirds of defaulters report either a substantial decrease in income or a layoff, unemployment, or reduced work hours in the past couple years; this would imply more conservatively that Δ_{gb} is too low by a factor of roughly 1.5. But as discussed in Low (2021) this is likely a lower bound for the appropriate adjustment factor, because unemployment or reduced work hours may often be mild compared to the other liquidity shocks also triggering default. Thus the best adjustment factor is likely somewhere between these numbers, so I set $\Delta_{gb} = 1.75 \cdot .0218 = .0381$.[29]

As calibrated, disaster shocks are substantially more common in this model than in most others. Many models do not even allow for large income shocks. Laufer (2018) allows for an unemployment shock, and notes that (at the time) doing so was very unusual for the literature. Some more recent papers (e.g. Campbell et al. (2020)) allow for very realistic income processes, but I am not aware of any other model that allows for liquidity shocks besides income shocks. Since these kinds of shocks do indeed trigger many defaults (Ganong and Noel, 2021; Low, 2021), accounting for them will improve the accuracy of the model.

The calibrated values for Δ_{gb} and other income parameters described above are used in both $\mathcal{AWD}$ and $\mathcal{UWD}$. It will be shown later that disaster shocks drive more default in $\mathcal{AWD}$ than in $\mathcal{UWD}$ and that this difference has important policy implications. Since both versions of the model have the same income process, these differences are not driven by different income processes; they instead arise because $\mathcal{AWD}$ generates realistic abovewater default rates and $\mathcal{UWD}$ does not.

Liquid assets: The interest rate R on risk-free, liquid assets is set to 1. ξ is set to .185, following Kaplan and Violante (2014).

Preferences: The risk aversion parameter γ is set to a common value of 2. The discount factor β is also set is to a common value of .94.

Recall that υ_1 determines the strength of the bequest motive, while υ_2 controls the extent to which bequests are a luxury good. In a typical lifecycle model, agents without a bequest motive would optimally consume most or all of their assets before death. Empirically, many people do leave very small bequests, but many also leave large ones. A typical calibration strategy for a lifecycle

[29] Ganong and Noel (2021) provide evidence for a very similar adjustment factor. They find that one-third of abovewater defaulters have no decrease in income, while roughly half have either no decrease in income or only a very mild one. Thus to account for non-income liquidity shocks Δ_{gb} should be adjusted upward by a factor between 1.5 and 2.

model is therefore to set υ_1 to match the size of bequests in the data, and υ_2 to match their frequency. However, as discussed in Section 5, this model generates frequent and large bequests that align well with the evidence but depend little on the bequest motive. This is because the model calibration yields psychic moving costs that are often extremely high in order to match the very low moving rates of longtime homeowners. Many equity-rich homeowners optimally avoid paying these psychic moving costs by staying in their homes until they die, leaving large bequests even if bequest motives are quite weak.[30] Since υ_1 and υ_2 are poorly identified, I set them to 1 and 15 respectively for technical reasons.[31] These generate a weaker bequest motive that in many other models. The model produces very similar results for a wide range of different values for these parameters.

The elasticity of substitution between consumption and housing, θ, is set to a standard value of .5 (Li et al., 2016).[32]

The consumption deflator is the median OECD equivalence scale, calculated from PSID data, by age.

Housing: The flow value of housing, κ, is set to a typical value of 7.5%.

Property taxes ζ_t are set to 1% (Kaplan et al., 2020); house maintenance costs ζ_m are set to 1.5% (Ngai and Sheedy, 2019). Proportional buying costs ϕ_b are set to 3% to account for closing fees and moving costs. Proportional selling costs ϕ_s are set to 7.5%, which allows for the typical 6% brokers' fees as well as 1.5% for closing fees etc.

Psychic moving costs: The maximum of the Uniform ψ distribution for new home buyers at age 23, Ψ^I_{23}, is set to .5, so the distribution has a mean and median of .25. There is no exact analog to Ψ^I_{23} in available data, but setting Ψ^I_{23} to .5 approximates the median psychic moving costs for young (or not rooted) homeowners in Koşar et al. (2021). Thereafter, I assume Ψ^I_t increases linearly until reaching Ψ^I_{64}, which is set internally. It is constant thereafter, during retirement.

[30] In the model, agents can extract equity without selling their home by refinancing a standard mortgage, but because mortgages amortize to period T, significant equity extraction late in life leads to large mortgage payments and is likely to violate the PTI constraint for retired agents. The model does not allow agents to take out reverse mortgages; in reality, very few homeowners do. The surprisingly low demand for reverse mortgages by equity-rich but cash-poor homeowners remains an active area of research. See Moulton and Haurin (2019) for a discussion.

[31] The model's code is better behaved if $\upsilon_1 > 0$ so that the marginal utility of wealth upon death is positive. Conditional on being positive, υ_1 matters very little, so I set it to 1 for simplicity. υ_2 needs to be larger than the borrowing limit so that the bequest function is defined for agents who die in debt, and 15 turns out to be large enough to serve this purpose.

[32] See Li et al. (2016) for a review of the literature on θ. Cross-sectional estimates of θ are typically substantially below one, because households spend more on housing in areas where housing is more expensive. But time-series estimates of θ are often well above one, because the correlation between aggregate expenditures on housing and the aggregate price of housing is weak or even negative. It seems likely that psychic moving costs could help explain this disconnect, because with them few households will find it optimal to move even if $\theta < 1$ and house prices change. Understanding if this is the case could be an important topic for future research.

Mortgages: I set the interest rate on mortgages, R_B, to 3.66% in order to match the median real after-tax interest rate on mortgages in the PSID assuming a 25% tax bracket.

Mortgage borrowers face a range of down payment requirements in reality, ranging from nearly 0% to 20%, depending heavily on their credit history and other factors. I set the exogenous LTV cap to 5%.

The exogenous PTI cap is set so that a homeowner's mortgage payment m can be no higher than 35% of income used for underwriting $\overline{y}$. As implemented in the model, this cap does not account for other debt payments and so is a "front-end" PTI limit. 35% is generous for a front-end limit; typical underwriting standards before the housing boom required a front-end PTI of 28% (Greenwald, 2017).

The financial costs of refinancing Φ_R are set to 2% of the value of the new mortgage, a fairly typical value (Hurst and Stafford, 2004).

Foreclosure: For homeowners in foreclosure in $\mathcal{AWD}$, the probability of sale as a function of relative list price is taken from Guren (2018). However, estimates from Guren (2018) are for the probability of sale within 13 weeks. The timing assumption in the model is that delinquent homeowners have just received the foreclosure auction notice. Typically, these notices arrive somewhere between two weeks and two months before the auction. Assuming that the foreclosure notice provides two months' notice and that it takes two weeks to list the house for sale gives a delinquent homeowner in the model roughly 6 weeks to sell the property. Therefore, letting $\pi_{Guren}(p^l)$ denote the probability of sale as a function of relative list price p^l from Guren (2018), the probability of sale as a function of list price in the model is $\pi_s(p^l) = 1 - ((1 - \pi_{Guren}(p^l)^{\frac{1}{13}})^6)$.[33]

In $\mathcal{UWD}$, $\pi_s(p)$ is not set to match the data but is set exogenously to 1. Therefore $\mathcal{UWD}$ reproduces the standard prediction that effectively abovewater homeowners sell to escape foreclosure.

Recall that in the model R_m^d represents the interest rate on delinquent mortgage debt, combining factors that make delinquency more expensive (such as delinquency fees and penalty interest) with factors that make it less expensive (such as the possibility of obtaining a loan modification). Unfortunately, there are no reliable estimates of R_m^d that I am aware of. Theoretically, $\mathcal{AWD}$ could set R_m^d internally, but it is not strongly identified because agents' delinquency and default decisions depend much more on psychic moving costs than they do on interest rates. For simplicity, I set R_m^d to be equal to the interest rate on current mortgage debt R^m, which implies that delinquency

[33] Decreasing the list price does increase the probability of sale, but the relationship between list price and probability of sale is concave; substantially reducing the list price increases the probability of sale only slightly. Guren (2018) establishes detailed empirical support for this concavity and develops a model that provides microfoundations for it. Anenberg and Kung (2018) also find a similar concave relationship between list price and probability of sale. Because the tradeoff between list price and probability of sale is poor, in the model only 0.07% of listed properties in foreclosure are listed for below market value. At any plausible list price, the probability of sale within 6 weeks is not high, which is well-established in the literature. Ngai and Sheedy (2019) target an average time-to-sell of 6.5 months, and provide an excellent discussion of the various time-to-sell estimates in the literature.

fees and loan modifications both have no effect in the model. So that defaulting does impose some financial cost, I set the accelerated rate at which unmaintained properties depreciate R_ζ to R_m^d. As shown in Table 5 in Section 5, with these parameter values $\mathcal{AWD}$ predicts a cure rate from foreclosure that is close to correct but slightly too low.

Recall that χ denotes the deadweight loss of foreclosure, as a fraction of the value of the foreclosed home. Typically, this parameter is chosen to match the discount foreclosed properties sell for at foreclosure auctions. But this neglects the carrying and resale costs mortgage servicers incur to offload a foreclosed property, and it is important for this paper not to overestimate the incentives abovewater homeowners have to default. Thus I set $\chi = .355$, which accounts for these costs, following Guren et al. (2020).

Foreclosure flags typically stay on credit records for seven years, starting from the date of the first missed payment. The effect they have on credit access is large at first and diminishes over time. As an approximation I set the length of the binary foreclosure flag to four years.

Externally calibrated parameters, along with their intrepretations, values, and sources are summarized in Table 2.

4.2 Second Stage

There are nine remaining parameters that I set internally so that the models match targeted moments. All parameters are calibrated using $\mathcal{AWD}$, except for the psychic cost of underwater foreclosure ψ_{F} in $\mathcal{UWD}$. ψ_{F} is set in $\mathcal{UWD}$ so that the aggregate foreclosure rate in $\mathcal{UWD}$ matches that in $\mathcal{AWD}$.[34]

These parameters, their targeted moments, and the resulting parameter values are summarized in Table 3. The fit of the calibrated models to the data is discussed in Section 5.

Table 3 contains the first major result of this paper. Given the strong financial incentives underwater borrowers have to default, underwater default rates are puzzingly low.[35] Few existing models even attempt to match underwater default rates, but the few that do need very high psychic default costs to do so: Ganong and Noel (2021) and Hembre (2018) set psychic default costs equal to 25% and 70% (respectively) of future consumption to explain why underwater default is so rare. Foote and Willen (2018) describe this mismatch between theory and evidence on underwater default as one of two "central" questions in the literature. $\mathcal{AWD}$ also targets the underwater default rate in

[34]The advantage of calibrating only one parameter specifically for $\mathcal{UWD}$ is that only $\pi_s(l = p)$ and ψ_{F} differ between $\mathcal{AWD}$ and $\mathcal{UWD}$, so it is easy to understand why their results differ. A potential disadvantage of this approach is that, because the other parameters are not calibrated separately for $\mathcal{UWD}$, $\mathcal{UWD}$ could have a poorer fit to targeted moments than $\mathcal{AWD}$. In practice, as shown in Section 5, the fit of the two models to targeted moments is nearly identical.

[35]For example, see Foote et al. (2008), Deng and Quigley (2013), Bhutta et al. (2017), and Gerardi et al. (2018).

Table 2: Externally Calibrated Parameters

Parameter	Interpretation	Value	Source
	Liquid Assets		
β	Discount factor	.94	Standard
R	Interest rate	1	Standard
ξ	Borrowing limit	.185	Kaplan and Violante (2014)
	Preferences		
γ	Risk aversion	2	Standard
υ_1	Strength of bequest motive	1	Not identified
υ_2	Extent to which bequests are luxury goods	15	Not identified
	Housing		
κ	Flow value of housing	.075	Standard
ζ_t	Property taxes	.01	Kaplan et al. (2020)
ζ_m	House maintenance costs	.015	Ngai and Sheedy (2019)
ϕ_b	House buying costs	.03	Standard
ϕ_s	House selling costs	.075	Standard
	Mortgages		
R_m	Mortgage interest rate	3.66	PSID
$LTV^{\max}$	Maximum LTV at origination	95	Standard
$PTI^{\max}$	Maximum PTI at origination	35	Standard
Φ_R	Refinancing costs	.02	Standard
	Foreclosure		
$\pi_s(l=p)$ ($\mathcal{AWD}$)	Probability of sale at market value	.267	Guren (2018)
$\pi_s(l=p)$ ($\mathcal{UWD}$)	Probability of sale at market value	1.	N/A
R_m^d	Interest rate on delinquent mortgage debt	3.66	R^m
χ	Deadweight loss of completed foreclosure	.355	Guren et al. (2020)

Note: Table shows the parameters calibrated outside the model, along with their interpretations, values, and sources.

Table 3: Internally Calibrated Parameters

Parameter	Interpretation	Target	Value
		Preference Parameters	
ω	Utility weight on nonhousing consumption	Mean housing expenditures over income	0.8
		Housing Parameters	
σ_p	Std. dev. of house prices	Underwater rate	0.02
Ψ_l^S	Location parameter for Cauchy ψ shocks	Moving hazard rate profile, <35	0.01
Ψ_s^S	Scale parameter for Cauchy ψ shocks	Moving hazard rate profile, <35	0.07
$\overline{\Psi}_{64}^I$	Max ψ for home buyer at $t=64$	Moving hazard rate profile, 35+	1.0
π_{NB}	% of initial nonhomeowners that cannot buy home	Homeownership profile	0.4
π_{nb}	Prob. no buying restriction removed	Homeownership profile	0.02
		Beliefs	
$\hat{\delta_{GB}}$	Perceived probability of disaster shock	% HtM homeowners	0.002
		Psych. Effectively Underwater (EUW) Foreclosure Costs	
ψ_{F}	Psych. EUW foreclosure cost ($\mathcal{AWD}$)	UW foreclosure rate	0.1
ψ_{F}	Psych. EUW foreclosure cost ($\mathcal{UWD}$)	Foreclosure rate in $\mathcal{AWD}$	0.02

Note: Table shows the parameters calibrated within the model, along with their interpretations, empirical targets, and values. The correspondence between parameters and their targets is for intuition only. Formally, every moment helps identify every parameter. HtM denotes "hand-to-mouth", which in the model is interpreted to mean nonpositive cash on hand. UW denotes "underwater" and EUW denotes "effectively underwater".

calibration, and is able to match it with psychic default costs of 0.1 (8.99% of future consumption).[36] This is still a large cost, but it is only about 13% of the costs in Hembre (2018) and 36% of the costs in Ganong and Noel (2021). The main reason $\mathcal{AWD}$ can match underwater default rates with psychic default costs that are so much smaller is because it has psychic moving costs that are large on average. Indeed in $\mathcal{AWD}$ psychic underwater default costs are only about 13% of average psychic moving costs for underwater homeowners. Thus, within the model, psychic moving costs explain about 89% of the underwater default rate puzzle.[37]

Many other models of default have even smaller default costs (e.g Demyanyk et al. (2017), Schelkle (2018), Hu (2019), Krivenko (2019), Campbell et al. (2020), Guren et al. (2020), Kaplan et al. (2020)). These models, like $\mathcal{UWD}$, generate mostly underwater default but are calibrated to match aggregate default rates; matching aggregate default rates without abovewater default requires much more underwater default than is in the data and and therefore low default costs. Relative to these papers, one contribution of $\mathcal{AWD}$ is to match *underwater* default rates with (comparatively) low default costs.

5 Model Fit

This section discusses the fit of $\mathcal{AWD}$ and $\mathcal{UWD}$ to the data. It does so in three parts. First, it discusses the empirical fit of the models in dimensions besides default and foreclosure. Second, it explores agents' default decisions in the two versions of the model, to put later results into context. Finally, it discusses the fit of the models to data related to defaults and foreclosures.

5.1 Fit of Models to Non-Default Moments

5.1.1 Standard Moments

Figure 4a plots the homeownership rate over the lifcycle, in data from the PSID and in the two versions of the model. Both models match the homeownership profile very well. This is important because mobility rates as a function of time in the home are targeted in calibration to discipline the process governing ψ, which in turn drives several results in the paper. Lifecycle considerations have

[36] Besides psychic moving costs, there are several other reasons underwater default rates might be lower than models predict. For example, homeowners might view strategic default as morally wrong (Guiso et al., 2013), might not know they are underwater (Chan et al., 2016a), might worry about lender recourse (Ghent and Kudlyak, 2011), might be unable to afford a move, etc.

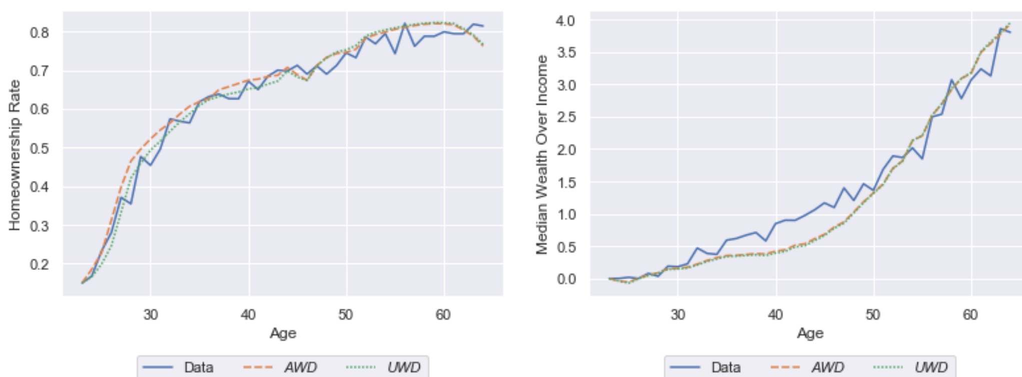
[37] Laufer (2018) also allows for both psychic moving costs and psychic default costs, and he needs default costs of 29% *and* moving costs of 54% of future consumption to match underwater default rates. Thus in his model psychic moving costs also explain the majority of the low underwater default rate puzzle. It is not entirely clear why he still needs psychic default costs to be so much higher than in this paper. One likely factor is that many underwater homeowners in his model expect house prices to keep falling and yet still do not default.

important effects on homeowner mobility rates, so the model must match when agents buy homes to meaningfully target when they move out of homes.

Figure 4b plots median wealth as a fraction of income over the lifecycle, which is not targeted during calibration, in data from the PSID and in the two versions of the model. The two models produce very similar wealth over income profiles that match the data fairly well, though they somewhat underpredict median wealth around age 45.

Figure 4: Homeownership and Wealth by Age

(a) Homeownership Rate by Age **(b)** Median Wealth Over Income by Age



NOTE: Figure shows homeownership rate (left panel) and median household wealth over income (right panel) by age, in $\mathcal{AWD}$, in $\mathcal{UWD}$, and in data from the PSID.

Table 4 shows the fit of the models to other targeted moments not directly related to mortgage default. Overall both models fit the data well. Lower underwater foreclosure costs ψ_F in $\mathcal{UWD}$ make high-leverage mortgages more attractive, and so (with the same standard deviation of house prices σ_p) more homeowners are underwater. This means $\mathcal{UWD}$ needs somewhat lower underwater default rates than it would otherwise need to match aggregate default rates.

Table 4: Model Fit to Non-default Moments

TARGET	SOURCE	EMPIRICAL VALUE	$\mathcal{AWD}$	$\mathcal{UWD}$
% homeowners HtM	Kaplan and Violante (2014)	42.4%	42.1%	42.5%
% mortgagors UW	1998 & 2001 SCF	2.5%	2.5%	3.1%
Median $\frac{\text{housing expenses}}{\text{income}}$	PSID	0.224	0.224	0.224

NOTE: Table shows the fit of the two versions of the model to targeted moments. HtM denotes "hand-to-mouth", which in the model is interpreted to mean nonpositive cash on hand. UW denotes underwater.

5.1.2 Homeowner Mobility

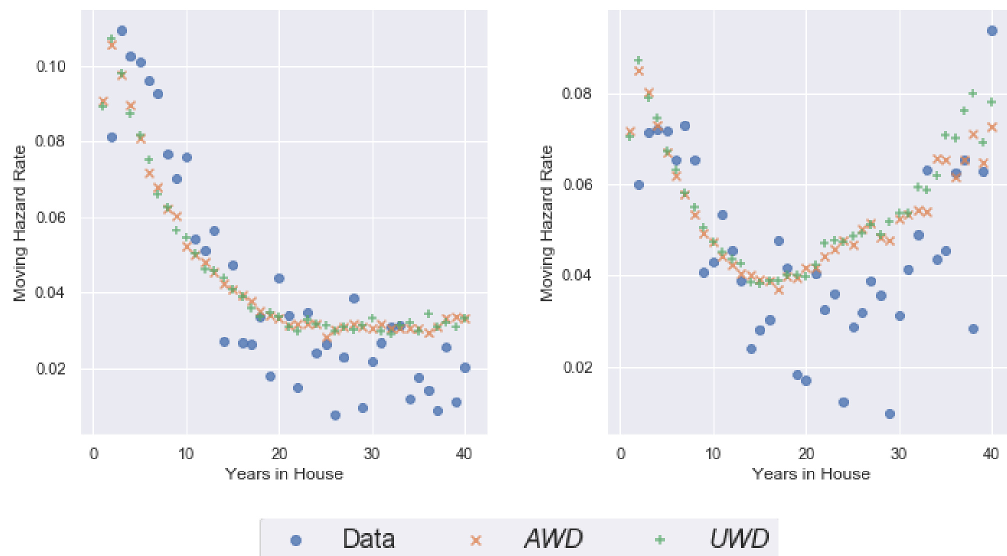
Recall that without psychic moving costs, the model would predict that mobility should be low for new homeowners and high for longtime homeowners, the opposite of the pattern seen in the data. This mismatch between the base theory and the data identifies the scale parameter Ψ_s^S and the location parameter Ψ_l^S of the shocks to ψ. Without shocks to ψ, new homeowners rarely move, because their homes provide nearly-optimal levels of housing consumption. The calibrated model generates moves for new homeowners through negative ψ shocks, which identifies Ψ_s^S. Since shocks to ψ can also be positive, this generates a strong selection effect; homeowners who receive negative ψ shocks move, while those with positive ψ shocks stay, so the homeowners who remain are selected to have high ψ. However, this force on its own is not enough to match the low mobility rates of longtime homeowners, who have houses that provide increasingly suboptimal levels of housing consumption. The additional growth in ψ that the model to do so needs identifies Ψ_l^S.

Together, Ψ_s^S and Ψ_l^S determine how ψ evolves with time in a home. However, in reality psychic moving costs may also evolve with age. To separate age and time effects on ψ, the model allows the initial ψ distribution for new homeowners to change with age, as determined by $\overline{\Psi}_{64}^I$. Higher values of $\overline{\Psi}_{64}^I$ lead to higher initial values of ψ for older home buyers. During calibration, $\overline{\Psi}_{64}^I$ is identified by the difference in moving hazard rates as a function of time in the home between homeowners who bought their homes before age 35 (when "young") and those who bought their homes after age 35 (when "old"). Hypothetically, if young and old homeowners had very similar moving hazard rates, that would be an indication that ψ varied little with age. Conversely, if old homeowners had moving hazard rates that were lower than those of young homeowners, that would be an indication that ψ increases with age and not just time in the home.

Figure 5 plots moving hazard rates as a function of age at purchase and time in a home, in data from the AHS and in the two versions of the model. Both models are able to match moving hazard rates for agents who bought homes before age 35 quite closely. The fit for agents who bought their homes later in life is also close, although not quite as close. The models slightly overpredict mobility rates for new owners who bought their homes late. In theory, the models could lower these mobility rates by further increasing $\overline{\Psi}_{64}^I$. But higher initial values of ψ make homeownership less attractive, so if $\overline{\Psi}_{64}^I$ is too high agents cease buying homes late in the lifecycle, which worsens the fit of the models to homeownership data. Indeed, this force explains the dip in homeownership before retirement seen in Figure 4a.

Figure 5: Moving Hazard Rate by Age at Purchase and Time in Home

(a) Age at Purchase < 35 **(b)** $35 \leq$ Age at Purchase ≤ 65



NOTE: Figure shows the percent of homeowners who move out of their home as a function of time in the home, in $\mathcal{AWD}$, in $\mathcal{UWD}$, and in data from the the American Housing Survey.

5.1.3 Psychic Moving Costs

This paper argues that psychic moving costs are key to understanding homeowners' default decisions, so it is important that psychic moving costs in the calibrated models be approximately correct. Unfortunately, direct data on psychic moving costs are almost impossible to find, and typically the closest available substitute is actual mobility decisions. Hence this paper follows others, including Kennan and Walker (2011), Bayer et al. (2016), Bartik (2018), Ngai and Sheedy (2019) and Oswald (2019), in calibrating psychic moving costs to match mobility decisions in a structural model.[38]

However, in a unique survey Koşar et al. (2021) directly elicit psychic moving costs using counterfactual choice probabilities, and so they provide direct evidence with which to compare the psychic moving costs in this paper. In Table 7 of their paper, they report that mean psychic moving costs to move to an identical home in the same neighborhood are 16%, 29%, and 162% of income for "mobile","stuck", and "rooted" respondents, respectively. These numbers reflect the income raises that respondents are willing to forgo to avoid moving, not one-time payments.[39] Unfortunately these

[38]To the best of my knowledge, this is the first paper to calibrate psychic moving costs to move *anywhere* using a consumption-savings model with more than one house size. This is an important innovation, because it is precisely the disconnect between what a standard such model would predict (mobility rates that increase with time in the home) and the data (mobility rates that decrease with time in the home) that provides identification.

[39]It is worth emphasizing just how large these numbers are; even respondents who self-identify as able and willing to move (i.e. "mobile") require on average an increase in lifetime income of 16% to move to an identical home in the

numbers combine both renters and homeowners, so to approximate the numbers for homeowners I adjust them upwards by the ratio of median psychic moving costs for homewners to that of renters given by Table 8, yielding psychic moving costs equal to 21%, 42%, and 194% for mobile, stuck, and rooted homeowners respectively. Combining mobile and stuck homeowners (with weights given by Table 2) yields psychic moving costs of 24.4% for not rooted homeowners.

To compare the calibrated models to these numbers, I first compute homeowners' Willingness-to-Pay (WTP) to avoid moving expressed in terms of permanent income raises agents are willing to forgo to avoid moving.[40] Because the results reported in Koşar et al. (2021) exclude the approximately 12.6% of homeowners with moving costs so high they cannot be estimated ("never-movers"), I drop the 12.6% highest WTPs generated in the model. For the remaining homeowners ("ever-movers") I calculate a cutoff value $\overline{WTP}$, defining agents to be "rooted" if and only if they have $WTP > \overline{WTP}$ and setting $\overline{WTP}$ so that the fraction of rooted ever-mover homeowners in my simulated data is equal to the fraction of rooted ever-mover homeowners in Koşar et al. (2021). The mean WTP to avoid moving for "not rooted" ever-mover homeowners in $\mathcal{AWD}$ is 33.6%, and it is 168.2% for "rooted" homeowners.[41] Given the many methodological differences between Koşar et al. (2021) and this paper, the similarity between their estimates and mine is encouraging and provides a strong indication that the distribution of homeowners' willingness to move in the model is approximately correct.

5.1.4 Bequests

Recall that Section 4.1 sets the parameters of the bequest function for numerical convenience, and not to match data on bequests. Yet it still matches the data fairly well. $\mathcal{AWD}$ generates an aggregate bequests-to-wealth ratio of 0.021 (versus .009 in De Nardi and Yang (2014)), a ratio of median wealth at age 75 to median wealth at age 50 of 1.18 (versus 1.51 in Kaplan et al. (2020)), and a 90th percentile of bequest over income of 6.21 (versus 4.53 in De Nardi and Yang (2014)). In a general lifecycle model with weak bequest motives, agents optimally consume most or all of their wealth before dying. However, this model generates frequent and large bequests that match the data well even with essentially no bequest motive because many agents optimally avoid paying psychic moving costs by staying in their homes until they die.

While the precise explanation for empirical bequest patterns is clearly outside the scope of this

same neighborhood.

[40] Because of methodological differences between Koşar et al. (2021) and this paper, this WTP is more comparable to their psychic moving costs than ψ is. Consider for example an agent with very high ψ whose home was far smaller than she would like, and who as a result would be very willing to move despite the high ψ. Such an agent would be willing to move in many hypothetical scenarios and so would have low estimated psychic moving costs in Koşar et al. (2021); in my model this agent would have a high ψ but low WTP to avoid moving.

[41] The numbers are nearly identical in $\mathcal{UWD}$, at 33.7% and 168.1% respectively.

paper, it is notable that psychic moving costs – introduced to more accurately model mortgage default – also appear to help the model match the empirical distribution of bequests. This finding is especially intriguing since psychic moving costs may help explain some puzzles in the bequest literature, e.g. households without children leave similar bequests to those with children, parents typically do not transfer bequests to their children until death, and homeowners leave much larger bequests than renters (Barczyk et al., 2019). This demonstrates that psychic moving costs likely have important implications in many settings besides mortgage default, so the strategy to calibrate psychic moving costs in this paper should be valuable in many other contexts.

5.2 Default and Foreclosure

In order to put later results on delinquency and foreclosure into context, this subsection discusses how and why default occurs in the two versions of the model.

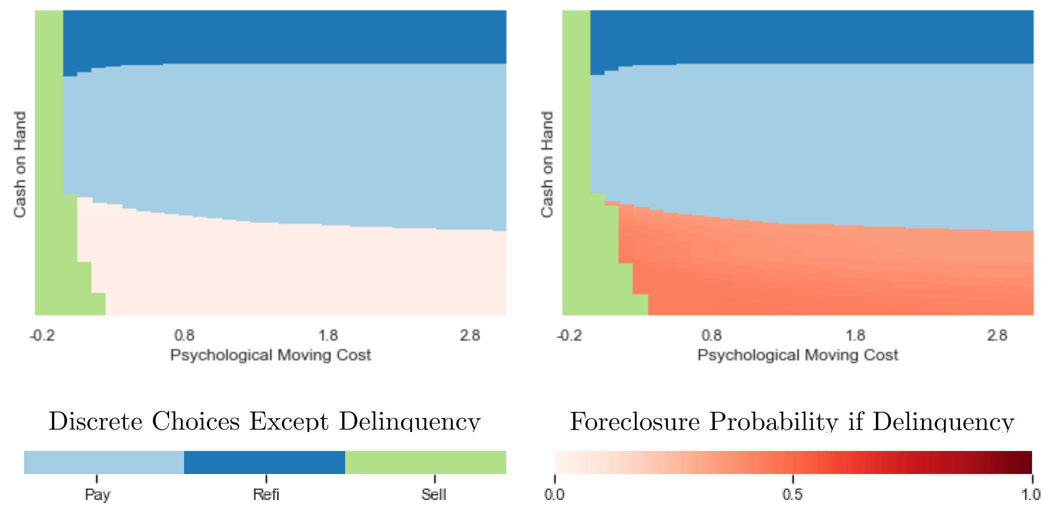
Figure 6a shows the discrete choices, and their implied next-period foreclosure probabilities, as a function of ψ (x-axis) and cash-on-hand (y-axis) for an example abovewater homeowner with an LTV of 79 in the disaster income state in $\mathcal{UWD}$. Because the homeowner has equity, for low values of ψ (on the left of the figure) she sells her home to extract the equity. For higher levels of ψ, moving is unattractive, but if she has little cash-on-hand (the lower center and right in the figure) then paying the mortgage is also unattractive, so she chooses delinquency.[42] This lets her skip her mortgage payment while still providing her with a high chance of keeping her home. In $\mathcal{UWD}$ this option is particularly attractive, because even if her income does not recover and she cannot keep her home, she will still be able to sell her home and extract her equity; her actual risk of foreclosure is very nearly zero. Thus in $\mathcal{UWD}$ abovewater homeowners become delinquent at fairly high rates but this leads to foreclosure very rarely and only if the property depreciates enough to turn equity effectively negative.

Figure 6b shows the discrete choices, and their implied next-period foreclosure probabilities, for a homeowner with the same state variables in $\mathcal{AWD}$. The most notable difference in Figure 6b is that in $\mathcal{AWD}$ delinquency can lead to foreclosure, even though the homeowner has equity. This raises the expected cost of delinquency, and so the region in which the homeowner chooses delinquency is somewhat smaller. However, the probability of foreclosure conditional on delinquency is still substantially below one; the homeowner might reinstate her mortgage if her income recovers, and may otherwise be able to sell. Also note that, while higher values of ψ make delinquency more attractive relative to selling, they also make paying more attractive relative to delinquency. The

[42] Note she does not refinance to extract her equity, because her mortgage payment is high enough that a cashout refinance with an even higher payment would violate the PTI constraint.

Figure 6: Abovewater Homeowner Choices and Implied Foreclosure Probabilities

(a) $\mathcal{UWD}$ **(b)** $\mathcal{AWD}$



Notes: Figure shows the discrete choice (if not becoming delinquent) and the implied foreclosure probability (if becoming delinquent) for a homeowner at age 28, with an LTV of 79 and the disaster income shock. The x-axis represents the psychic moving cost ψ. The y-axis represents cash-on-hand. The left panel is for $\mathcal{UWD}$; the right panel is for $\mathcal{AWD}$.

effect of ψ on default risk in $\mathcal{AWD}$ is not monotonic.

Figure 7a shows the same objects for the same homeowner in $\mathcal{UWD}$, except increasing her mortgage payment so that she is underwater with an LTV of 106. Because she is underwater, instead of selling at low values of ψ she now chooses delinquency.[43] This delinquency is "strategic" in the sense that it occurs even if she has enough cash-on-hand to pay her mortgage, and in the sense that she will not get current even if her income recovers so foreclosure occurs with probability near one. At higher levels of ψ, delinquency more closely resembles classic "double-trigger" default: it only occurs at lower levels of cash-on-hand, and the homeowner is likely to reinstate her mortgage if her income recovers despite being underwater.

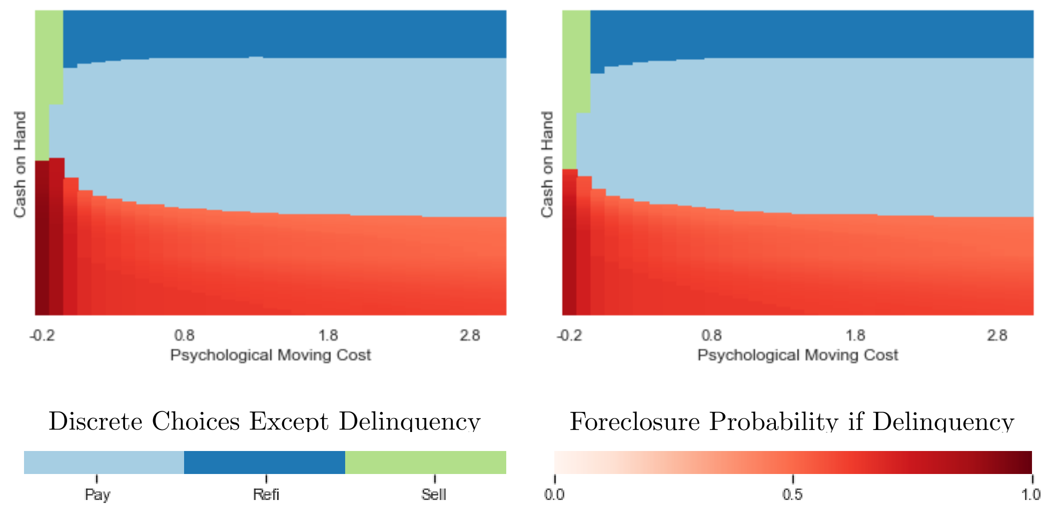
Figure 7b shows the same information as Figure 7a except in $\mathcal{AWD}$. Because $\mathcal{AWD}$ targets a lower underwater foreclosure rate than that produced by $\mathcal{UWD}$, it has a higher psychic cost of underwater default. As a result, the region of "strategic" delinquency in Figure 7b is smaller, and partly replaced by "double-trigger" default.

Finally, a comparison of Figures 6b and 7b demonstrates the theoretical benefit of including psychic moving costs in a model of mortgage default. Empirically, abovewater default rates are higher and underwater default rates are lower than existing models can explain. In $\mathcal{AWD}$, high

[43] At low levels of ψ but very high levels of cash-on-hand, she prefers to sell her home even at a loss to preserve her access to mortgage credit and to avoid paying the psychic cost of underwater foreclosure.

Figure 7: Underwater Homeowner Choices and Implied Foreclosure Probabilities

(a) $\mathcal{UWD}$ **(b)** $\mathcal{AWD}$



Notes: Figure shows the discrete choice (if not becoming delinquent) and the implied foreclosure probability (if becoming delinquent) for a homeowner at age 28, with an LTV of 106 and the disaster income shock. The x-axis represents the psychic moving cost ψ. The y-axis represents cash-on-hand. The left panel is for $\mathcal{UWD}$; the right panel is for $\mathcal{AWD}$.

values of ψ generally make abovewater default more likely and underwater default less likely. Psychic moving costs therefore help $\mathcal{AWD}$ explain both puzzles simultaneously.

5.3 Fit of Models to Foreclosure Data

Table 5 displays the fit of the models to the data for foreclosure-related moments. Note that moments in the table are not targeted in calibration, except that psychic effectively-underwater foreclosure costs are set so that $\mathcal{AWD}$ matches the underwater foreclosure rate (ignoring the causal effect of delinquency on equity to more closely resemble the data in Foote et al. (2008)) and $\mathcal{UWD}$ matches the aggregate foreclosure rate in $\mathcal{AWD}$.

Since $\mathcal{AWD}$ targets the underwater foreclosure rate during calibration, it matches this moment closely. Because $\mathcal{UWD}$ generates little abovewater default, it overstates the underwater foreclosure rate by nearly a factor of three in order to match the aggregate foreclosure rate. Given that $\mathcal{AWD}$ does not target foreclosure starts, or the various ways foreclosure starts can end, its success matching these moments is notable. In $\mathcal{UWD}$ borrowers who become delinquent can always sell to escape foreclosure if they want to, so they enter foreclosure and escape foreclosure through selling at rates both roughly double that in the data.[44] Both models overstate the mortgage chargeoff rate relative

[44] A prominant prediction of existing theories is that all abovewater homeowners should sell their homes before

Table 5: Model Fit to Foreclosure-Related Moments

Moment	Empirical Source	Empirical Value	$\mathcal{AWD}$ Value	$\mathcal{UWD}$ Value
"UW" foreclosure rate	Foote et al. (2008)	2.53%	2.54%	6.96%
Foreclosure rate	Mortgage Bankers' Association (1998-2001)	0.45% - 0.5%	0.45%	0.45%
Foreclosure starts	Mortgage Bankers' Association (1998-2001)	1.2%	1.18%	2.58%
In foreclosure ⇒ current rate	Herkenhoff and Ohanian (2019)	57.5%	53.7%	64.5%
In foreclosure ⇒ foreclosed rate	Herkenhoff and Ohanian (2019)	34.6%	37.8%	17.1%
In foreclosure ⇒ sales rate	Herkenhoff and Ohanian (2019)	7.9%	8.5%	18.4%
Mortgage chargeoff rate	FRED (1998-2001)	0.16%	0.21%	0.27%
Third-party foreclosure sales	FHFA (2015-2018)	15-33%	11.1%	0.0%

Note: Table shows the fit of the two versions of the model to empirical moments related to foreclosure. Moments in the table are not targeted in calibration, except $\mathcal{AWD}$ is calibrated to match the underwater foreclosure rate and $\mathcal{UWD}$ is calibrated to match the foreclosure rate in $\mathcal{AWD}$. Because the data in Foote et al. (2008) do not account for the negative causal effect of delinquency on equity, in the model for this purpose "underwater" is defined using equation Equation 8 and not Equation 9. See Figure 17 in the online appendix of Herkenhoff and Ohanian (2019) for moments from that paper. In the model, the mortgage chargeoff rate is the ratio of total lender losses from foreclosure to total mortgage debt outstanding. The third-party foreclosure sales rate is the percent of foreclosures that occur when the borrower's LTV at foreclosure is below $100(1-\chi_D)$.

to the data, though the miss by $\mathcal{AWD}$ is less than half as large as the one by $\mathcal{UWD}$. Similarly, both models underestimate the number of third-party foreclosure sales, but again $\mathcal{AWD}$ comes much closer to matching the data.[45] This may indicate that even $\mathcal{AWD}$ overstates the relationship between equity and foreclosure.

To investigate the relationship between equity and foreclosure in more detail, Figure 8a plots foreclosure rates as a function of LTV (normalized by foreclosure rates at an LTV of 80) in the two models as well as in the data from Foote et al. (2008), adjusted for measurement error and the fact that Foote et al. (2008) use data from a recourse state.[46] To make the comparison between $\mathcal{AWD}$ and the data clearer, Figure 8b plots the same information but excludes results from $\mathcal{UWD}$. Note the two figures have different y-axes.

Foote et al. (2008) estimate that homeowners with an LTV of 120 are roughly five times more likely to experience foreclosure than those with an LTV of 80.[47] In $\mathcal{UWD}$, they are more than 80 times more likely to experience foreclosure, because $\mathcal{UWD}$ overstates the underwater default rate and because only measurement error can generate any measured default at an LTV of 80. By comparison, $\mathcal{AWD}$ matches the data much more closely with a ratio of roughly eight to one. Again, even $\mathcal{AWD}$ appears to overstate the relationship between equity and default for underwater homeowners. For abovewater homeowners, $\mathcal{AWD}$ matches the relationship between equity and default quite closely.

Models of mortgage default are frequently used to understand the effect on foreclosure rates of events or policies that affect equity, such as house price declines or caps on mortgage LTVs

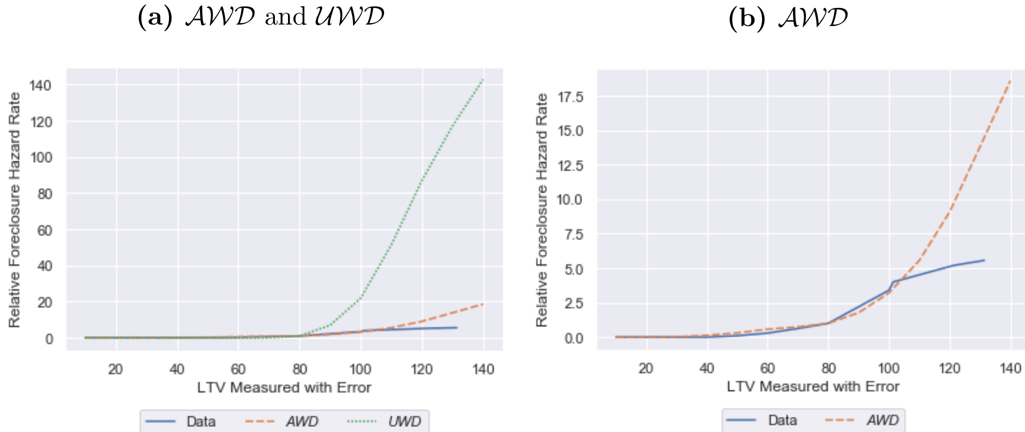
foreclosure (Foote and Willen, 2018). To the best of my knowledge, $\mathcal{UWD}$ is the first model to make specific predictions about *when* in the foreclosure process they should sell.

[45] According to conventional wisdom, mortgage lenders bid what they are owed at foreclosure auctions. If a third party wins the auction, that is an indication that the lender did not lose money from the foreclosure. In this model, this occurs when the borrower's LTV at foreclosure is below $100(1-\chi_D)$.

[46] The figure assumes that in the models property values are measured with classical error, which in percentage terms has mean 0 and standard deviation 10. These numbers approximate measurement error in typical data (Molloy and Nielsen, 2018). However, empirically measurement error in property values appears to be much more Berkson than classical and to lead to much less attenuation bias than classical measurement error does. See Low (2021).

[47] Estimates from other papers are similar. See footnote 10.

Figure 8: Relative Foreclosure Hazard as a Function of Equity



NOTES: Figure plots foreclosure hazard rates as a function of LTV, normalized by foreclosure rates at an LTV of 80. Data are foreclosure hazard rates in Figure 3 in Foote et al. (2008). To account for measurement error in Foote et al. (2008), foreclosure hazard rates from the models assume that property values are measured with classical error, which in percentage terms has mean 0 and standard deviation 10. Foote et al. (2008) use data from Massachusetts, which is a recourse state. Ghent and Kudlyak (2011) estimate that, for underwater homeowners, the effect of recourse on foreclosure rates is equivalent to lowering the LTV by 8.6. Therefore, for LTVs above 100 empirical hazard rates in the figure are taken for LTVs 8.6 higher than that shown.

at origination. In particular, a very large portion of the literature has studied the relationship between the drop in house prices and the increase in foreclosures during the Great Recession. One significant contribution of this paper is to develop a model that matches the empirical relationship between negative equity and default more closely than standard models. It is therefore likely to give more accurate answers to questions involving the relationship between negative equity and default, examples of which are studied in Section 6.3.

6 Model Results

This section first considers the role of negative equity and adverse liquidity shocks in generating default in $\mathcal{AWD}$ and $\mathcal{UWD}$ to put later policy experiments into context. Then it considers the role of psychic moving costs ψ in driving default. Finally, it conducts two sets of policy experiments. The first is focused on the role of negative equity in driving default, and the second is focused on the role of adverse shocks.

6.1 Adverse liquidity shocks, negative equity, and default

Much of the mortgage default literature studies the role of negative equity and adverse liquidity shocks in "triggering" default. The two dominant theories of default – strategic and double-trigger default – differ in whether adverse life shocks trigger default but agree that negative equity does. The main theoretical contribution of $\mathcal{AWD}$ is to generate default triggered by adverse shocks alone, and not by negative equity. However, strategic and double-trigger default also occur in both $\mathcal{AWD}$ and $\mathcal{UWD}$, and the relative significance of each kind of default is a quantitative question important for interpreting the policy experiments that follow. Moreover, Foote and Willen (2018) describe understanding the relative contributions of negative equity and adverse shocks to default the second "central" question in the literature, so understanding how $\mathcal{AWD}$ and $\mathcal{UWD}$ answer this question differently places the theoretical contributions of $\mathcal{AWD}$ into context.

Table 6 shows the percent of homeowners, defaulters, and foreclosures that are underwater, abovewater but effectively underwater, and effectively abovewater.[48] The numbers for homeowners are important. Negative equity makes default substantially more likely in $\mathcal{AWD}$, as it does in virtually every model of default. The comparatively low abovewater default rate is still quantitatively important because a large majority of homeowners have equity. Table 6 shows that in the model effectively abovewater homeowners outnumber underwater homeowners by about 40 to one. Double-trigger homeowners are even rarer; just 0.3% of homeowners have both negative equity and the disaster shock. The rarity of double-trigger homeowners limits the explanatory power of double-trigger default in both $\mathcal{AWD}$ and $\mathcal{UWD}$, even though double-trigger homeowners are quite likely to default.

Because default and foreclosure take place over two periods, the role of shocks in triggering default is somewhat subtle. Just 14.4% of defaulters in $\mathcal{AWD}$ are underwater at the time of delinquency, but 28.7% are underwater at the time of foreclosure. Part of this increase is due to the fact that a negative house price shock after delinquency makes foreclosure more likely. However, part of this increase is due to default causing negative equity rather than vice-versa; of underwater foreclosures in $\mathcal{AWD}$, just 49.9% would be underwater if they had made their mortgage payment the previous period. The label "$\mathcal{UWD}$" denotes that most defaulters are underwater in $\mathcal{UWD}$, but not all; in $\mathcal{UWD}$ 28.8% of defaulters have positive effective equity at the time of default, even though negative effective equity is a necessary condition for foreclosure. This number is so high in $\mathcal{UWD}$ in part because abovewater homeowners become delinquent at very high rates, expecting to be able to sell to avoid foreclosure if necessary. Some of them are unlucky and experience a negative house price shock the next period, which drives their equity effectively negative and prevents them from selling. Others are driven effectively underwater by the causal negative effect of delinquency on equity. Still, negative equity plays a much less important role in driving default in $\mathcal{AWD}$ than it does in $\mathcal{UWD}$ or other more standard models; in $\mathcal{AWD}$ 43.9% of defaulters have positive effective equity at the

[48] Recall that default is used in this paper to mean delinquency that leads to foreclosure. The numbers for defaulters and foreclosures therefore represent the same agents, but at different time periods.

Table 6: % of Homeowners with Disaster Shock and Equity

	Homeowners		Defaulters		Foreclosures	
	$\mathcal{AWD}$	$\mathcal{UWD}$	$\mathcal{AWD}$	$\mathcal{UWD}$	$\mathcal{AWD}$	$\mathcal{UWD}$
All						
LTV $<$ 92.5	93.1	92.9	73.5	28.8	43.9	0.0
92.5 $\leq$ LTV $\leq$ 100	4.6	4.2	12.2	22.3	27.3	26.9
LTV $>$ 100	2.4	2.9	14.4	48.9	28.7	73.1
No Disaster Shock						
LTV $<$ 92.5	85.9	85.8	1.9	0.5	7.3	0.0
92.5 $\leq$ LTV $\leq$ 100	4.0	3.7	0.4	11.9	4.8	2.8
LTV $>$ 100	2.1	2.6	7.2	41.4	9.0	38.9
Disaster Shock						
LTV $<$ 92.5	7.2	7.1	71.5	28.4	36.6	0.0
92.5 $\leq$ LTV $\leq$ 100	0.5	0.5	11.8	10.5	22.5	24.1
LTV $>$ 100	0.3	0.3	7.2	7.4	19.7	34.2

Note: Table shows the percent of homeowners, defaulters, and foreclosures in each LTV category overall (upper panel), without the disaster shock (middle panel), and with the disaster shock (lower panel). Defaulters are homeowners becoming delinquent this period who will lose their home to foreclosure next period. Homeowners with LTVs between 92.5 and 100 are abovewater, but because of transaction costs ϕ_s would lose money by selling their home, and so are effectively underwater.

time of foreclosure, while in $\mathcal{UWD}$ precisely 0.0% do.

The disaster shock triggers the vast majority of default in $\mathcal{AWD}$, in part because it is essentially the only force that can trigger abovewater default.[49] It is also particularly important in triggering underwater default. This is because $\mathcal{AWD}$ calibrates the psychic cost of effectively underwater foreclosure ψ_{F} to match the low underwater default rate. Double-trigger homeowners have very strong incentives to default and ψ_{F} is not large enough to deter them from doing so. The incentives for strategic default are typically weaker, and so ψ_{F} matches the underwater default rate largely by setting the strategic default rate appropriately. This leads to strategic default being quite rare in $\mathcal{AWD}$: just 7.2% of defaulters have negative equity but no disaster shock, and another 0.4% have positive equity, but negative effective equity, and no disaster shock. However, $\mathcal{UWD}$ has a different calibration strategy and sets ψ_{F} to match the aggregate default rate. There are not enough double-trigger homeowners for it to do so, so it sets the strategic default rate much higher; thus

[49] 1.9% of defaulters in $\mathcal{AWD}$ have positive effective equity but not the disaster shock. While too rare to affect the results quantitatively, these intially puzzling defaults are interesting theoretically and are discussed in Section 6.2.

in $\mathcal{UWD}$ 41.4% of defaulters have negative equity but no disaster shock, and another 11.9% have positive equity, but negative effective equity, and no disaster shock.[50] Overall, adverse shocks play a significantly more important role in driving default in $\mathcal{AWD}$ than they do in $\mathcal{UWD}$, even though both models have the same income process.

6.2 Psychic moving costs and default

To further understand the causes and implications of default in the model, this subsection focuses on the role of psychic moving costs ψ. Empirically, Koşar et al. (2021) document substantial heterogeneity in homeowners' psychic moving costs, and allowing for this heterogeneity in a structural model of default is a key innovation of this paper. Table 7 shows the percent of defaulters that experienced a negative ψ shock immediately before defaulting, the mean ψ of defaulters, and the percent of defaulters with a negative ψ, for the various groups of defaulters studied in Section 6.1 in $\mathcal{AWD}$. To interpret values of ψ, recall that it is a penalty that multiplies both current and future utility in the event of a move. For example, a homeowner with $\psi = .5$ that is forced to move to an identical home at no financial cost suffers a 50% lifetime utility loss.

Table 7: Psychic Moving Costs and Default in $\mathcal{AWD}$

	% with $\downarrow \psi$	Mean ψ	% with $\psi < 0$
No Disaster Shock			
LTV < 92.5	28.4	1.8	0.0
$92.5 \leq$ LTV ≤ 100	90.9	0.0	81.8
LTV > 100	55.8	0.0	46.8
Disaster Shock			
LTV < 92.5	21.2	0.8	0.0
$92.5 \leq$ LTV ≤ 100	32.2	0.5	9.6
LTV > 100	38.7	0.4	19.4

Note: Table shows the percent of defaulters in each category that experienced a ψ decline the period of delinquency (first column). It also shows the mean ψ of defaulters (second column), and the percent of defaulters with negative ψ (third column). Defaulters are homeowners becoming delinquent this period who will lose their home to foreclosure next period. Homeowners with LTVs between 92.5 and 100 are abovewater, but because of transaction costs ϕ_s would lose money by selling their home, and so are effectively underwater.

[50] These defaults are "strategic" in the sense of occurring without a disaster shock. However, "strategic" default is sometimes interpreted as maximizing wealth, which these defaults do not necessarily do. Homeowners may choose to default on an underwater mortgage, even if doing so is not wealth-maximizing, because the house is not the appropriate size or because psychic moving costs are negative.

There is no precise and universal definition of strategic default in the literature, but it is often modeled as the wealth-maximizing exercise of a put option on a financial asset (Foote and Willen, 2018). Table 7 shows that in this paper strategic default is quite different and often involves a second "trigger" of its own in the form of a negative ψ shock. In this paper, many homeowners who default with negative equity and without an income shock do not sell their home because of their negative equity, as in standard theory, but they choose not to pay their mortgage because an outside shock (e.g. a job offer in a new location) has made moving attractive.[51] This is especially true for abovewater but effectively underwater homeowners, who have little financial incentive to default rather than pay. Negative shocks to ψ are a major reason why $\mathcal{AWD}$ still needs significant psychic costs of underwater foreclosure ψ_F to match the underwater default rate, even with psychic moving costs that are on average quite large. Note the large number of strategic defaulters with negative ψ implies that policies that prevent strategic default impose a welfare cost on homeowners in $\mathcal{AWD}$ that is absent in traditional models.[52]

Conversely, ψ is on average high when default is triggered by the adverse shock: 0.4 for underwater homeowners and 0.8 for effectively abovewater ones. Partly this is because ψ is large on average for all homeowners, but for abovewater defaulters there is an additional selection effect because homeowners with low ψ sell rather than default if they cannot pay their mortgage. These high psychic moving costs imply that policies that would allow these agents to stay in their homes would have higher welfare benefits for consumers than they would in models with $\psi = 0$.

It may seem surprising that there are any effectively abovewater defaulters with no disaster shock in Table 7. There are very few of them (as shown earlier in Table 6 they comprise 1.9% of defaulters) and quantitatively they play little role in the results, but their extremely high mean value of $\psi = 1.8$ is notable. These homeowners do not pay their mortgage because a permanent income shock or retirement has rendered their mortgage unaffordable in the long term; unlike other abovewater homeowners choosing to become delinquent, they do not have a reasonable chance of keeping their home. Yet they do not sell their home because of very high psychic moving costs, which make delaying a move by one period worth the considerable expected financial loss involved in defaulting rather than selling.[53] This is a psychic version of the "free rent" benefit of defaulting

[51] Empirically, the relationship between negative equity and mobility is an active area of research. For a recent paper on the topic with a useful review of the literature, see Bernstein (Forthcoming). Interestingly, the debate in the literature is whether or not negative equity *reduces* mobility, providing further evidence that classical wealth-maximizing strategic default is quite rare. This could also suggest that binding liquidity constraints may prevent underwater homeowners from defaulting after moving shocks, in much the same way that liquidity contraints sometimes prevent bankruptcy (Gross et al., 2014) and mortgage refinancing (Defusco and Mondragon, 2020).

[52] When solving the model numerically, ψ is capped below at -.2 because this is low enough to virtually always induce homeowners to move (either through selling or defaulting), and homeowner mobility is used to discipline ψ. Further lowering the minimum ψ changes homeowner mobility in the model very little, but has a significant effect on the psychic moving costs of strategic defaulters.

[53] In reality, homeowners may be able to avoid this kind of default by listing their house for sale and agreeing with

that is well-known in the literature, which is typically viewed as the financial benefit of avoiding rent payments for a period of time before foreclosure. Interestingly, the psychic value of "free rent" is negative for many strategic defaulters because they have $\psi < 0$, as shown in Table 7.

Finally, Table 8 presents the same information as Table 7, but for $\mathcal{UWD}$. The results are largely the same, with two exceptions. First, abovewater defaulters look much more similar to underwater defaulters in $\mathcal{UWD}$ than in $\mathcal{AWD}$, which is unsurprising since in $\mathcal{UWD}$ they must be effectively underwater at the time of foreclosure.[54] Second, ψ is typically somewhat higher for strategic defaulters in $\mathcal{UWD}$ because psychic foreclosure costs ψ_F are lower, and so underwater homeowners default more readily.

Table 8: Psychic Moving Costs and Default in $\mathcal{UWD}$

	% with $\downarrow \psi$	Mean ψ	% with $\psi < 0$
No Disaster Shock			
LTV < 92.5	83.6	0.1	69.1
92.5 ≤ LTV ≤ 100	65.7	-0.0	41.9
LTV > 100	44.0	0.1	18.6
Disaster Shock			
LTV < 92.5	23.6	0.5	0.2
92.5 ≤ LTV ≤ 100	34.9	0.5	10.5
LTV > 100	37.8	0.4	14.8

Note: Table shows the percent of defaulters in each category that experienced a ψ decline the period of delinquency (first column). It also shows the mean ψ of defaulters (second column), and the percent of defaulters with negative ψ (third column). Defaulters are homeowners becoming delinquent this period who will lose their home to foreclosure next period. Homeowners with LTVs between 92.5 and 100 are abovewater, but because of transaction costs ϕ_s would lose money by selling their home, and so are effectively underwater.

6.3 House price shocks and lender recourse

Section 6.1 notes that abovewater default is important in part because negative equity is rare. However, much of the literature studies the effect of an aggregate house price decline (especially the one that occured during the Great Recession) on foreclosures. If aggregate house prices fall, more homeowners will be underwater; indeed, during the Great Recession, negative equity was common

the buyer to delay the transaction for a year. This kind of default is too rare in the model to justify complicating it in this way. But to avoid artificially increasing the amount of abovewater default in the model, I only count defaults that occur before age 78, since the incentives behind this kind of default are particularly strong immediately before agents die with certainty at age 85.

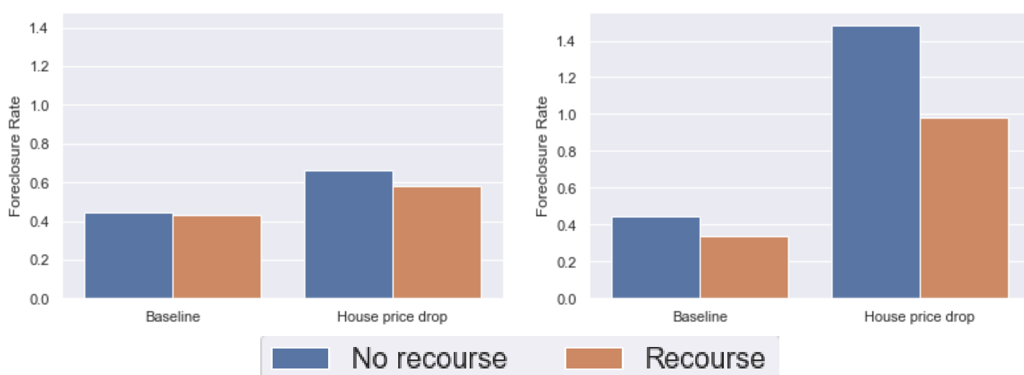
[54] For example, the median LTV of abovewater defaulters with no disaster shock is 53.6 in $\mathcal{AWD}$ and 91.5 in $\mathcal{UWD}$.

and most defaulters were underwater.

To understand the contributions of this paper in scenarios when negative equity is more common, this subsection considers the effect on foreclosure rates of a one-time, unexpected decrease in all house prices by .1 log points (roughly 9.5%). It also considers the effect of a policy aimed at discouraging underwater default, called "lender recourse", which allows lenders to sue defaulters for the difference between the balance on the mortgage and the value of the foreclosed property.[55] Recourse is implemented in the models by allowing lenders to costlessly seize the minimum of half the borrower's assets after foreclosure and the difference between the outstanding mortgage balance and the value of the property at foreclosure (for the foreclosure value function associated with recourse, see equation 35 in Section A.3). Figure 9 presents aggregate foreclosure rates in $\mathcal{AWD}$ and $\mathcal{UWD}$, introducing the aggregate house price decline in the models both with and without recourse.

Figure 9: Foreclosure Rates With a Decline in House Prices and Recourse

(a) $\mathcal{AWD}$ **(b)** $\mathcal{UWD}$



Notes: Figure shows foreclosure rates under various scenarios. The house price decline is a drop in house prices of .10 log points for every homeowner. Recourse allows lenders to costlessly seize the minimum of half the borrower's assets after foreclosure and the difference between the outstanding mortgage balance and the value of the property at foreclosure. Because foreclosure occurs the period after default, the figure shows foreclosure rates the period after the decline in house prices.

Recall from Figure 8 that the relationship between equity and foreclosure risk is more empirically accurate in $\mathcal{AWD}$ than it is in $\mathcal{UWD}$. It is also much weaker, so the aggregate drop in house prices induces a much smaller increase in foreclosures. Without recourse, the foreclosure rate increases from 0.45% to 0.67% in $\mathcal{AWD}$; the increase is 4.7 times larger in $\mathcal{UWD}$, where it increases to 1.48%. Thus accounting for abovewater default is important for conducting counterfactuals in which equity levels change.

Without the house price decline, recourse has almost no effect on foreclosure rates in $\mathcal{AWD}$.

[55] Recourse is prohibited in many states but allowed in many others. Even states that theoretically allow recourse differ considerably in how practical it actually is. For a detailed discussion of recourse laws by state, see Ghent and Kudlyak (2011).

Part of the reason is that in $\mathcal{AWD}$ many defaulters still have equity at the time of foreclosure, and so cannot be sued even with recourse because the property is worth more than the mortgage balance. However, as shown in Table 6, 28.7% of foreclosed homeowners in $\mathcal{AWD}$ are underwater and would be subject to recourse. Recourse does little to prevent these defaults because few of them are strategic. If the delinquent homeowners could become current, they would. However, with an aggregate decline in house prices, more default is strategic in $\mathcal{AWD}$ and so recourse is more effective; it lowers the foreclosure rate from 0.67% to 0.58%.

Many previous theoretical papers have studied the role of lender recourse in the mortgage market. Because these models generate little abovewater default, they either predict that recourse lowers both underwater and aggregate default rates (Li et al. (2014), Hatchondo et al. (2014), Campbell and Cocco (2015), Corbae and Quintin (2015), Gete and Zecchetto (2018a), Gete and Zecchetto (2018b)) or neither (Quintin (2012), Mitman (2016).) Because it generates substantial abovewater default, $\mathcal{AWD}$ is the first model in the literature to match the more subtle evidence that recourse lowers underwater default rates (Ghent and Kudlyak (2011), Demiroglu et al. (2014), Dobbie and Goldsmith-Pinkham (2015), Chan et al. (2016b)) but not aggregate default rates (Clauretie (1987), Ghent and Kudlyak (2011), Desai et al. (2013), Li and Oswald (2014)) except after large house price declines (Jones (1993), Chan et al. (2016b)). The unique ability of $\mathcal{AWD}$ to match this evidence provides further confidence in its conclusions.

Because default is more strategic in $\mathcal{UWD}$, recourse is more effective at preventing default both before and after the drop in house prices. Because recourse is more effective in $\mathcal{UWD}$, and because the drop in house prices causes more foreclosures in $\mathcal{UWD}$, after the drop in house prices recourse prevents 5.8 times more foreclosures in $\mathcal{UWD}$ than in $\mathcal{AWD}$. The quantitatively large differences between $\mathcal{AWD}$ and $\mathcal{UWD}$ in the effects of recourse again emphasize the importance of accounting for abovewater default when conducting counterfactuals related to equity levels.

6.4 Adverse liquidity shocks and forbearance

Much of the literature focuses on double-trigger default, which – like abovewater default in $\mathcal{AWD}$ – requires an adverse liquidity shock, so the differences between standard models and $\mathcal{AWD}$ in the role of adverse shocks in generating default may be unclear. Moreover, the COVID-19 pandemic raises important questions about appropriate policy responses to widespread financial hardship when negative equity is extremely rare; in 2020 tens of millions of Americans became unemployed and yet essentially no homeowners were underwater after years of strong house price growth.[56]

[56] See https://libertystreeteconomics.newyorkfed.org/2021/09/if-prices-fall-mortgage-foreclosures-will-rise/.

To understand the contributions of this paper in scenarios when adverse shocks are common, this subsection considers the effect on foreclosure rates of an unexpected, one-time income shock transitioning 10% of agents without the disaster shock to the disaster shock, with no change in house prices.[57] It also introduces two stylized versions – one "limited" and one "generous" – of forbearance, a policy allowing homeowners to temporarily suspend their mortgage payments without penalty that became far more widespread during the COVID-19 pandemic.

In the model, both versions of forbearance are unexpectedly offered to delinquent homeowners. Both versions allow delinquent homeowners until the end of the period to make their mortgage payments for both the previous and current period or to sell their home (for the value functions associated with forbarance, see Section A.3 beginning with Equation 36). The two versions of forbearance differ only in who forbearance is offered to. "Limited" forbearance is offered only to agents without the disaster shock, and so it can be interpreted as assistance targeted towards homeowners who may have had a sharp income drop the period of delinquency but now likely have the income to be able to keep their home with help. "Generous" forbearance is offered to all delinquent homeowners, regardless of their ability to make up their missed payments.[58] Foreclosure rates in $\mathcal{AWD}$ and $\mathcal{UWD}$ with no forbearance, limited forbearance, and generous forbearance, with and without the aggretate drop in income, are shown in Figure 10.

The disaster state can trigger default in abovewater homeowners in $\mathcal{AWD}$. Thus without forbearance, the aggregate increase in the number of homeowners in the disaster state induces an increase in foreclosures that is 1.9 times greater in $\mathcal{AWD}$ than in $\mathcal{UWD}$. Thus policy interventions to reduce foreclosures may be significantly more justified after an aggregate income shock than a standard model would predict, even if negative equity is rare and house prices do not fall.

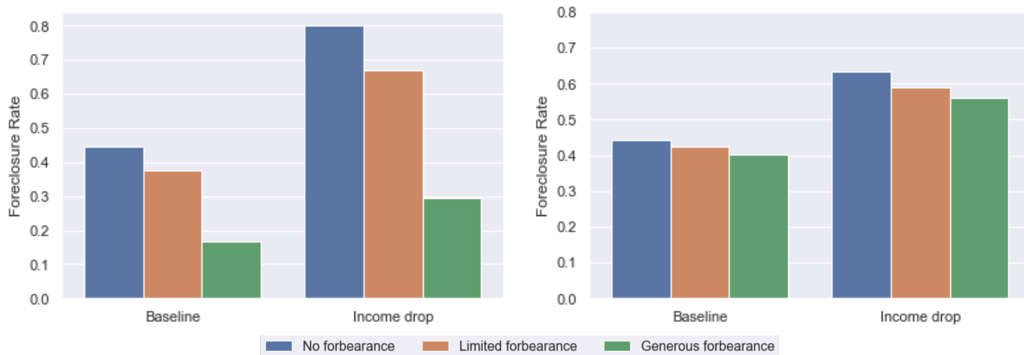
The difference between the two models becomes even more stark after introducing forbearance. The scope for limited forbearance to help distressed homeowners is not large; those in the disaster income state are not offered it, while those whose income has fully recovered do not need it. Still, limited forbearance can help delinquent homeowners without the disaster state escape foreclosure if their cash-on-hand is too low to get current even though their income has recovered. In $\mathcal{AWD}$, limited forbearance prevents foreclosures for abovewater homeowners in this group; in $\mathcal{UWD}$, they

[57] This shock occurs at the very beginning of the period, before the regular income shocks. The probability of a transition out of the disaster state Δ_{bg} is .537, so only 4.63% of agents without the disaster shock get it from the experiment and still have it after the regular shocks.

[58] Both versions of forbearance in the model are stylized and differ in important ways from the forbearance mandated by the 2020 CARES Act. In particular, borrowers entering forbearance in 2020 were typically allowed more time to make up their missed payments than the model allows them; I require repayment at the end of the period only because this is computationally tractable. Limited forbearance seems likely to provide a lower bound on the number of foreclosures avoided because of forbearance like that in the CARES Act, both because it only lasts until the end of the period and because it is not offered to many delinquent borrowers. Generous forbearance likely provides an upper bound on the foreclosures prevented by forbearance like that in the CARES Act because it allows *all* effectively abovewater borrowers to escape foreclosure either through becoming current or selling.

Figure 10: Foreclosure Rates With a Decline in Aggregate Income and Forbearance

(a) $\mathcal{AWD}$ **(b)** $\mathcal{UWD}$



NOTES: Figure shows foreclosure rates with and without an aggregate drop in income, under three different types of forbearance. The aggregate income shock transitions 10% of agents without the disaster shock to the disaster shock. Generous forbearance allows all homeowners in foreclosure until the end of the period to make their mortgage payments for both the previous and current period, either by paying the mortgage or by selling the home. Limited forbearance is restricted to homeowners not in the disaster income state. Because foreclosure occurs the period after default, the figure shows the change in foreclosure rates the period after the decline in income.

will escape foreclosure through selling anyway. Hence, after the aggregate shock, limited forbearance is 3.0 times more effective at preventing foreclosures in $\mathcal{AWD}$ than $\mathcal{UWD}$.

Generous forbearance is notably more effective than limited forbearance in $\mathcal{AWD}$, because it gives even homeowners who cannot afford to keep their homes the time to sell them before foreclosure. By contrast, there is almost no difference in effectiveness between limited and generous forbearance in $\mathcal{UWD}$, because in $\mathcal{UWD}$ giving homeowners more time to sell their homes does not affect the probability of sale. After the aggregate shock, limited forbearance is 7.0 times more effective at preventing foreclosures in $\mathcal{AWD}$ than $\mathcal{UWD}$. This experiment demonstrates that, by giving distressed homeowners more time to sell, there is a benefit to forbearance that standard models will miss.

7 Conclusion

Existing theories predict that abovewater homeowners nearly always sell their homes to avoid foreclosure, and so generate abovewater default rates substantially lower than in the data. This paper develops a model in which, after an adverse shock, some abovewater homeowners wager their equity in a gamble to keep their homes by choosing delinquency over selling. This wager can be attractive because the chance of avoiding foreclosure and psychic moving costs are both high, but it is still risky and some abovewater homeowners who become delinquent go on to lose their homes to foreclosure. Through this channel, the model generates empirically accurate levels of abovewater default.

By comparing the baseline version of the model to a more standard version of the model with low abovewater default rates, this paper shows that accounting for abovewater default helps address several important policy-relevant issues. It allows the model to answer both "central" (Foote and Willen (2018)) questions in the literature, and leads to a relationship between equity and default in the model that approximates the one in the data. It also leads the model to place more weight on adverse shocks, and less weight on negative equity, as triggers of mortgage default. This has many important implications in policy experiments. For example, the policy experiments suggest that lender recourse is much less effective at preventing foreclosures than standard models predict, while forbearance is much more effective.

This paper demonstrates the importance of abovewater default in general, but leaves many specific questions unaddressed. For example, the lifecycle aspects of the model allow homeowner mobility to be endogenous, which is critical for identifying psychic moving costs. However, because of the number of time periods that would be required, it is not feasible to calibrate the lifecycle model in this paper at a weekly or monthly frequency. This is an important limitation shared by almost all existing models. A higher-frequency model of abovewater default could address many important outstanding questions about the resolution of uncertainty regarding income shocks, the home sales process, and the foreclosure process.

Furthermore, Appendix A.2 shows that, for the results in this paper, it mostly does not matter whether search frictions prevent abovewater defaulters from selling to escape foreclosure or equity shocks that occur in delinquency do. However, the two classes of explanations for abovewater default have different policy implications and so disentangling them could be an important question for future research. If search frictions prevent abovewater homeowners from selling to avoid foreclosure, then giving delinquent homeowners more warning before a foreclosure auction could reduce foreclosure rates at low cost. However, if foreclosure fees instead drive abovewater default, then reducing foreclosure fees could prevent abovewater default and so help both borrowers and lenders.

A critical area for future research is the nature of psychic moving costs. A number of papers with varied data and methodology – including this one – estimate psychic moving costs to be on average extremely large. If these moving costs do in fact reflect actual consumer preferences, they imply much greater welfare gains from policies that prevent foreclosures or distressed sales than models without psychic moving costs would predict. But if psychic moving costs are in part behavioral, there may be an important role for early intervention to encourage distressed abovewater homeowners to sell rather than default.

Finally, the models studied here are partial equilibrium and do not capture the complex interactions between the mortgage market, the housing market, and the broader economy that are studied in many other papers. How abovewater default affects these kinds of interactions is an important question for future research. Moreover, the model is calibrated to be applicable in a "typical" time period, which limits its applicability to specific historical episodes, e.g. the 2007 financial crisis or the COVID-19 pandemic. General equilibrium models that are calibrated for these time periods and that generate empirically realistic levels of abovewater default will yield new and valuable insights.

References

An, Xudong, Larry Cordell, Liang Geng, and Keyoung Lee (2021a) "Inequality in the Time of COVID-19: Evidence from Mortgage Delinquency and Forbearance," *Working Paper*.

An, Xudong, Yongheng Deng, and Stuart A. Gabriel (2021b) "Default Option Exercise over the Financial Crisis and beyond," *Review of Finance*, pp. 153–187.

Anenberg, Elliot and Edward Kung (2018) "Interest Rates and Housing Market Dynamics in a Housing Search Model," *Working Paper*.

Attanasio, Orazio, Agnes Kovacs, and Patrick Moran (2020) "Temptation and Commitment: Understanding Hand-to-Mouth Behavior," *NBER Working Paper 27944*.

Barczyk, Daniel, Sean Fahle, and Matthias Kredler (2019) "Save, Spend, or Give? A Model of Housing, Family Insurance, and Savings in Old Age," *Working Paper*.

Bartik, Alexander W. (2018) "Moving Costs and Worker Adjustment to Changes in Labor Demand: Evidence from Longitudinal Census Data," *Working Paper*.

Bartik, Timothy J., J.S. Butler, and Jin-Tan Liu (1992) "Maximum score estimates of the determinants of residential mobility: Implications for the value of residential attachment and neighborhood amenities," *Journal of Urban Economics*, Vol. 32(2), pp. 233–256.

Bayer, Patrick, Robert McMillan, Alvin Murphy, and Christopher Timmins (2016) "A Dynamic Model of Demand for Houses and Neighborhoods," *Econometrica*, Vol. 84(3), pp. 893–942.

Bernanke, Ben S. (1983) "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression," *The American Economic Review*, Vol. 73(3), pp. 257–276.

Bernstein, Asaf (Forthcoming) “Negative Home Equity and Household Labor Supply,” *Journal of Finance*.

Bhutta, Neil, Jane Dokko, and Hui Shan (2017) “Consumer Ruthlessness and Mortgage Default during the 2007 to 2009 Housing Bust,” *Journal of Finance*.

Campbell, John Y., Nuno Clara, and João F. Cocco (2020) “Structuring Mortgages for Macroeconomic Stability,” *Working Paper*.

Campbell, John Y. and João F. Cocco (2015) “A Model of Mortgage Default,” *The Journal of Finance*, Vol. 70(4), pp. 1495–1554.

Capponi, Agostino, Ruizhe Jia, and David Aaron Rios (2021) “The Effect of Mortgage Forbearance on Refinancing: Evidence from the COVID-19 Period,” *Working Paper*.

Carroll, Christopher and Andrew Samwick (1997) “The Nature of Precautionary Wealth,” *Journal of Monetary Economics*, Vol. 40, pp. 41–71.

Chan, Sewin, Samuel Dastrup, and Ingrid Gould Ellen (2016a) “Do Homeowners Mark to Market? A Comparison of Self-Reported and Estimated Market Home Values During the Housing Boom and Bust,” *Real Estate Economics*, Vol. 44(3), pp. 627–657.

Chan, Sewin, Andrew Haughwout, Andrew Hayashi, and Wilbert Van Der Klaauw (2016b) “Determinants of Mortgage Default and Consumer Credit Use: The Effects of Foreclosure Laws and Foreclosure Delays,” *Journal of Money, Credit, and Banking*, Vol. 48(2-3), pp. 393–413.

Chatterjee, Satyajit and Burcu Eyingungor (2015) “A Quantitative Analysis of the U.S. Housing and Mortgage Markets and the Foreclosure Crisis,” *Review of Economic Dynamics*, Vol. 18, pp. 165–184.

Cherry, Susan F., Erica Xuewei Jiang, Gregor Matvos, Tomasz Piskorski, and Amit Seru (2021) “Government and Private Household Debt Relief during COVID-19,” *NBER Working Paper 28357*.

Clauretie, Terrence M. (1987) “The Impact of Interstate Foreclosure Cost Differences and the Value of Mortgages on Default Rates,” *Real Estate Economics*, Vol. 15(3), pp. 152–167.

Corbae, Dean and Erwan Quintin (2015) “Leverage and the Foreclosure Crisis,” *Journal of Political Economy*, Vol. 123 (1), pp. 1 – 65.

Currie, Janet and Erdal Turkin (2015) “Is There a Link between Foreclosure and Health?” *American Economic Journal: Economic Policy*, Vol. 7(1), pp. 63–94.

Davis, Morris and Stijn van Nieuwerburgh (2015) "Housing, Finance, and the Macroeconomy," in *Handbook of Regional and Urban Economics*, Vol. 5(12).

De Nardi, Mariacristina and Fang Yang (2014) "Bequests and heterogeneity in retirement wealth," *European Economic Review*, Vol. 72, pp. 182–196.

Defusco, Anthony A. and John Mondragon (2020) "No Job, No Money, No Refi: Frictions to Refinancing in a Recession," *The Journal of Finance*, Vol. 75(5), pp. 2327–2376.

Demiroglu, Cem, Evan Dudley, and Christopher M. James (2014) "State Foreclosure Laws and the Incidence of Mortgage Default," *The Journal of Law and Economics*, Vol. 57(1), pp. 225–280.

Demyanyk, Yuliya, Dmytro Hryshko, María Jose Luengo-Prado, and Bent E. Sørensen (2017) "Moving to a Job: The Role of Home Equity, Debt, and Access to Credit," *American Economic Journal: Macroeconomics*, Vol. 9(2), pp. 149–181.

Deng, Yongheng and John M. Quigley (2013) "Woodhead Behavior and the Pricing of Residential Mortgages," *IRES Working Paper Series.*

Desai, Chintal A., Gregory Elliehausen, and Jevgenijs Steinbuks (2013) "Effects of Bankruptcy Exemptions and Foreclosure Laws on Mortgage Default and Foreclosure Rates," *Journal of Real Estate Finance and Economics*, Vol. 47, pp. 391–415.

Diamond, Rebecca, Adam Guren, and Rose Tan (2020) "The Effect of Foreclosures on Homeowners, Tenants, and Landlords," *NBER Working Paper 27358.*

Dobbie, Will and Paul Goldsmith-Pinkham (2015) "Debtor Protections and the Great Recession," *Working Paper.*

Elul, Ron, Nicholas S. Souleles, Souphala Chomsisengphet, Dennis Glennon, and Robert Hunt (2010) "What "Triggers" Mortgage Default?" *The American Economic Review, Papers and Proceedings*, Vol. 100(2), pp. 490–494.

Farrell, Diana, Fiona Greig, and Chen Zhao (2021) "Did Mortgage Forbearance Reach the Right Homeowners? Income and Liquid Assets Trends for Homeowners during the COVID-19 Pandemic," *Working Paper.*

Foote, Christopher L., Kristopher Gerardi, and Paul. S. Willen (2008) "Negative Equity and Foreclosure: Theory and Evidence," *Journal of Urban Economics*, Vol. 64, pp. 234–245.

Foote, Christopher L. and Paul. S. Willen (2018) "Mortgage-Default Research and the Recent Foreclosure Crisis," *Annual Review of Financial Economics*, Vol. 10, pp. 59–100.

Foster, Chester and Robert Van Order (1984) "An Option-Based Model of Mortgage Default," *Housing Finance Review*, Vol. 3(4), pp. 351–372.

Francesco D'Acunto, Christoph K. Scheuch, Thomas Rauter and Michael Weber (2020) "Perceived Precautionary Savings Motives: Evidence from Fintech," *NBER Working Paper No. 26817.*

Fulford, Scott L. (2015) "The Surprisingly Low Importance of Income Uncertainty for Precaution," *European Economic Review*, Vol. 79, pp. 151–171.

Fuster, Andreas, Benedict Guttman-Kenney, and Andrew Haughwout (2018) "Tracking and Stress-Testing U.S. Household Leverage," *FRBNY Economic Policy Review*, Vol. September, pp. 35–63.

Fuster, Andreas and Paul S. Willen (2017) "Payment Size, Negative Equity, and Mortgage Default," *American Economic Journal: Economic Policy*, Vol. 9(4), pp. 167–191.

Ganong, Peter, Damon Jones, Pascal Noel, Diana Farrell, Fiona Greig, and Chris Wheat (2020) "Wealth, Rate, and Consumption Smoothing of Typical Income Shocks," *Working Paper.*

Ganong, Peter and Pascal J. Noel (2021) "Why do Borrowers Default on Mortgages? A New Method for Causal Attribution," *Working paper.*

Garriga, Carlos and Aaron Hedlund (2019) "Mortgage Debt, Consumption, and Illiquid Housing Markets in the Great Recession," *Working Paper.*

Gerardi, Kristopher, Kyle F. Herkenhoff, Lee E. Ohanian, and Paul S. Willen (2018) "Can't Pay or Won't Pay? Unemployment, Negative Equity, and Strategic Default," *Review of Financial Studies*, Vol. 31(3), pp. 1098–1131.

Gete, Pedro and Franco Zecchetto (2018a) "Distributional Implications of Government Guarantees in Mortgage Markets," *The Review of Financial Studies*, Vol. 31(3), pp. 1064–1097.

——— (2018b) "Mortgage Design and Slow Recoveries. The Role of Recourse and Default," *Working Paper.*

Ghent, Andra C. and Marianna Kudlyak (2011) "Recourse and Residential Mortgage Default: Evidence from US States," *The Review of Financial Studies*, Vol. 24(9), pp. 3139 – 3186.

Giannone, Elisa, Qi Li, Nuno Paix ao, and Xinle Pang (2020) "Unpacking Moving," *Working Paper.*

Greenwald, Daniel L. (2017) "The Mortgage Credit Channel of Macroeconomic Transmission," *Working Paper.*

Gross, Tal, Matthew J. Notowidigdo, and Jialan Wang (2014) "Liquidity Constraints and Consumer Bankruptcy: Evidence from Tax Rebates," *The Review of Economics and Statistics*, Vol. 96(3), pp. 431–443.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales (2013) "The Determinants of Attitudes Toward Strategic Default on Mortgages," *The Journal of Finance*, Vol. 68(4), pp. 1473–1515.

Guren, Adam M. (2018) "House Price Momentum and Strategic Complementarity," *Journal of Political Economy*, Vol. 126(3), pp. 1172–1218.

Guren, Adam M., Arvin Krishnamurthy, and Timothy J. McQuade (2020) "Mortgage Design in an Equilibrium Model of the Housing Market," *The Journal of Finance*, Vol. 76(1), pp. 113–168.

Han, Lu and William C. Strange (2015) "The Microstructure of Housing Markets: Search, Bargaining, and Brokerage," in J.V. Henderson G. Duranton and W.C. Strange eds. *Handbook of Regional and Urban Economics*.

Hatchondo, Juan Carlos, Leonardo Martinez, and Juan M. Sánchez (2014) "Mortgage Defaults and Prudential Regulations in a Standard Incomplete Markets Model," *Working Paper*.

Haughwout, Andrew, Donghoon Lee, Joelle Scally, and Wilbert van der Klaauw (2021) "Forbearance Participation Declines as Programs' End Nears," *Liberty Street Economics*, Available at https://libertystreeteconomics.newyorkfed.org/2021/08/forbearance-participation-declines-as-programs-end-nears/.

Head, Allen, Huw Lloyd-Ellis, and Chenggang Zhou (2020) "Indebted Sellers, Liquidity and Mortgage Standards," *Working Paper*.

Hedlund, Aaron (2016a) "The Cyclical Dynamics of Illiquid Housing, Debt, and Foreclosures," *Quantitative Economics*, Vol. 7, pp. 289–328.

——— (2016b) "Illiquidity and its Discontents: Trading Delays and Foreclosures in the Housing Market," *Journal of Monetary Economics*, Vol. 83, p. 2016.

Hembre, Erik (2018) "HAMP, Home Attachment, and Mortgage Default," *Working paper*.

Herkenhoff, Kyle F. and Lee E. Ohanian (2019) "The Impact of Foreclosure Delay on U.S. Unemployment," *Review of Economic Dynamics*, Vol. 31, pp. 63–83.

Hu, Malin (2019) "Default Risk Heterogeneity and Borrower Selection in the Mortgage Market," *Working Paper*.

Hurst, Erik and Frank Stafford (2004) "Home Is Where the Equity Is: Mortgage Refinancing and Household Consumption," *Journal of Money, Credit and Banking*, Vol. 36(6), pp. 985–1014.

Jeske, Karsten, Dirk Krueger, and Kurt Mitman (2013) "Housing, Mortgage Bailout Guarantees and the Macro Economy," *Journal of Monetary Economics*, Vol. 60, pp. 917–935.

Jones, Lawrence D. (1993) "Deficiency Judgments and the Exercise of the Default Option in Home Mortgage Loans," *Journal of Law and Economics*, Vol. 36, pp. 115–138.

Kaplan, Greg, Kurt Mitman, and Giovanni L. Violante (2020) "The Housing Boom and Bust: Model Meets Evidence," *Journal of Political Economy*, Vol. 128(9).

Kaplan, Greg and Giovanni L. Violante (2014) "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, Vol. 82(4), pp. 1199–1239.

Kennan, John and James R. Walker (2011) "The Effect of Expected Income on Individual Migration Decisions," *Econometrica*, Vol. 79(1), pp. 211–251.

Kermani, Amir and Francis Wong (2021) "Racial Disparities in Housing Returns," *Working Paper*.

Koşar, Gizem, Tyler Ransom, and Wilbert van der Klaauw (2021) "Understanding Migration Aversion Using Elicited Counterfactual Choices Probabilities," *Journal of Econometrics*.

Krivenko, Pavel (2019) "Unemployment and the US Housing Market during the Great Recession," *Working Paper*.

Laufer, Steven (2018) "Equity Extraction and Mortgage Default," *Review of Economic Dynamics*, Vol. 28, pp. 1–33.

Li, Wenli, Haiyong Liu, Fang Yang, and Rui Yao (2016) "Housing over Time and over the Life Cycle: A Structural Estimation," *International Economic Review*, Vol. 57(4), pp. 1237–1260.

Li, Wenli, Costas Meghir, and Florian Oswald (2014) "Consumer Bankruptcy and Mortgage Default," *Working Paper*.

Li, Wenli and Florian Oswald (2014) "Recourse and the Residential Mortgage Market: the Case of Nevada," *Working Paper*.

Low, David (2015) "Mortgage Default with Positive Equity," *Mimeo*.

——— (2020) "Estimating the Home Equity of Foreclosed Homeowners," *Mimeo*, available at https://www.dropbox.com/s/gwu9v7fb0z3mqgn/EstimatingHomeEquity.pdf?dl=0.

——— (2021) "What Triggers Mortgage Default? New Evidence from Linked Administrative and Survey Data," *Working Paper*.

Mitman, Kurt (2016) "Macroeconomic Effects of Bankruptcy & Foreclosure Policies," *American Economic Review*, Vol. 106(8), p. 2219–2255.

Mnasri, A. (2015) "Renting vs buying a home: A matter of wealth accumulation or of geographic stability?" *Journal of Economic Dynamics and Control*, Vol. 60, pp. 42–72.

Molloy, Raven and Eric Nielsen (2018) "How Can We Measure the Value of a Home? Comparing Model-Based Estimates with Owner-Occupant Estimates," *FEDS Notes*.

Moulton, Stephanie and Donald Haurin (2019) "Unlocking Housing Wealth for Older Americans: Strategies to Improve Reverse Mortgages," *Economic Studies at Brookings*.

Ngai, Rachel and Kevin D. Sheedy (2019) "The Decision to Move House and Aggregate Housing-Market Dynamics," *Working Paper*.

Olafsson, Arna and Michaela Pagel (2018) "The Liquid Hand-to-Mouth: Evidence from Personal Finance Management Software," *The Review of Financial Studies*, Vol. 31(11), pp. 4398–4446.

Oswald, Florian (2019) "The Effect of Homeownership on the Option Value of Regional Migration," *Quantitative Economics*, Vol. 10(4), pp. 1453–1493.

Piazzesi, Monika and Martin Schneider (2016) "Housing and Macroeconomics," in John B. Taylor and Harald Uhlig eds. *Handbook of Macroeconomics*, Vol. 1(2): Elsevier.

Quintin, Erwan (2012) "More Punishment, Less Default?" *Annals of Finance*, Vol. 8, pp. 427–454.

Riddiough, Timothy J. (1991) "Equilibrium mortgage default pricing with non-optimal borrower behavior," *Ph.D. Dissertation*.

Schelkle, Thomas (2018) "Mortgage Default During the U.S. Mortgage Crisis," *Journal of Money, Credit and Banking*, Vol. 50, pp. 1101–1137.

Vandell, Kerry D. (1995) "How Ruthless is Mortgage Default? A Review and Synthesis of the Evidence," *Journal of Housing Research*, Vol. 6(2), pp. 245–264.

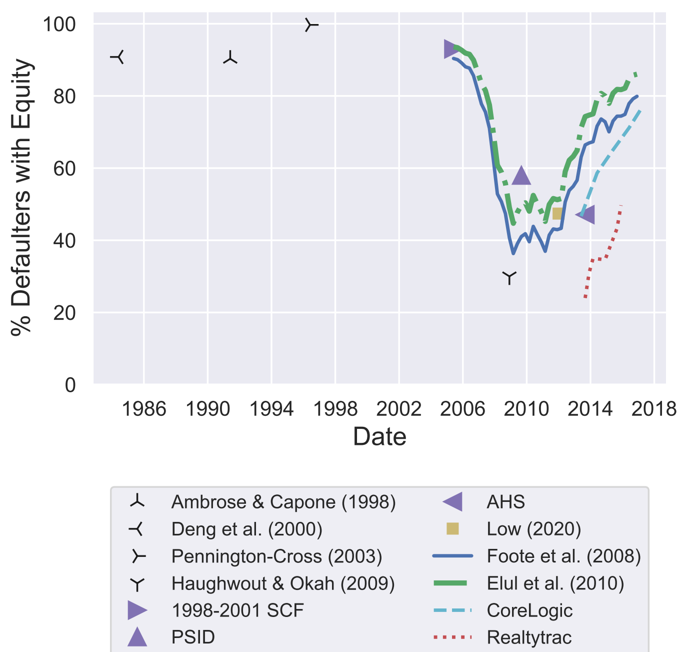
Venti, Steven F. and David A. Wise (1984) "Moving and housing expenditure: Transaction costs and disequilibrium," *Journal of Public Economics*, Vol. 23(1-2), pp. 207–243.

A Appendix

A.1 Evidence on Abovewater Default

There are a large number of existing sources on the fraction of defaulters that have positive equity. Figure 11 summarizes these sources by plotting the estimated percent of defaulters with positive equity against the year the data is from. The evidence consistently shows that most defaulters have equity in most years outside the 2007 financial crisis. This figure and its sources are discussed in detail in a companion empirical paper, Low (2020). Figure 1 in the online appendix to Ganong and Noel (2021) presents additional comparable evidence.

Figure 11: Evidence on the Home Equity of Foreclosed Homeowners



Notes: Figure shows on the y-axis estimates of the percent of homeowners with equity from various papers, surveys, and websites. The x-axis represents the year (or year midpoint, for date ranges) the estimate is from. Black tri-points denote estimates available in previous papers. Purple triangles denote estimates from survey data. Yellow square denotes estimate from Low (2020). Solid blue and green solid-dashed lines combine estimates from previous papers of default hazard rates as a function of equity with data on the distribution of equity provided to me by Fuster et al. (2018); I thank the authors for this data. Dashed and dotted lines denote estimates available from popular websites. This figure and its sources are discussed in detail in Low (2020).

The abovewater default seen in the data is sometimes attributed to measurement error. The evidence presented in Figure 11 comes from a wide variety of sources and yet is consistent, suggesting that measurement error does not play a major role in explaining empirical abovewater default rates. However, as discussed in detail in Low (2020), there are potential concerns with most evidence on this topic. For example, homeowners may systematically overestimate their home values in surveys, while home value estimates derived from local house price indices may miss idiosyncratic property depreciation. To address these potential concerns, Low (2020) implements a robust Bayesian Gibbs Sampling procedure to formally estimate the home equity of foreclosed homeowners and finds that between 2011 and 2013 roughly 47% of foreclosed homeowners had positive equity. This is the only formal robust estimate of the proportion of defaulters with equity in the literature, but it is remarkably consisent with all other evidence on the topic. Again this suggests that measurement error is an unlikely explanation for the abovewater default seen in the data.

Still, an important lesson from this paper is that abovewater default has several major policy-relevant theoretical implications. Further evidence on the equity of defaulters is therefore valuable. By showing that $\mathcal{AWD}$ performs better than $\mathcal{UWD}$ at matching underwater default rates, the foreclosure start rate, the escape rate from foreclosure by selling, the mortgage chargeoff rate, the third-party foreclosure sales rate (all in Table 4), the empirical relationship between equity and default (Figure 8), and the evidence on lender recourse (subsection 6.3), this paper provides further evidence that the abovewater default seen in the data is real and not due to measurement error.

A.2 An Alternative Interpretation of $\mathcal{AWD}$

The main body of the paper develops a model, $\mathcal{AWD}$, in which search frictions prevent some delinquent homeowners with equity from selling to avoid foreclosure. This subsection discusses an alternative interpretation of the model without search frictions, $\mathcal{AWD}2$, in which stochastic negative shocks to delinquent homeowners' equity perform the same role. These may be property depreciation shocks resulting from reduced property maintenance, foreclosure fees assessed by the lender, or stigma arising from the foreclosure process that makes it harder to sell a property in foreclosure.

$\mathcal{AWD}2$ is not necessarily intended to be quantitatively reasonable. It is not used in the main body of the paper in part because I am not aware of reliable evidence to discipline its parameters. Rather, $\mathcal{AWD}2$ is intended to produce the same results as $\mathcal{AWD}$ despite having a different interpretation. The contribution of $\mathcal{AWD}2$ is to provide intuition for which mechanisms in $\mathcal{AWD}$ are important for its results and which are not.

Recall that, in $\mathcal{AWD}$, a delinquent homeowner may list her home for sale at price $p^l \leq p$, in which case the property sells with probability $\pi_s(p^l)$. The only difference between $\mathcal{AWD}$ and $\mathcal{AWD}2$ is that, in $\mathcal{AWD}2$ the probability of a delinquent sale is one and $1 - \pi_s(p^l)$ instead represents the probability that the equity shock occurs and prevents the delinquent homeowner from selling. Therefore results in Section 6.4 that arise in $\mathcal{AWD}$ from forbearance giving abovewater homeowners more time to sell arise in $\mathcal{AWD}2$ from forbearance eliminating foreclosure fees and other equity shocks. With this interpretation, the dependence of $1 - \pi_s(p^l)$ on the list price p^l a homeowner would choose in $\mathcal{AWD}$ is awkward, but not important. In $\mathcal{AWD}$ the tradeoff between list price and probability of sale is so poor that only 0.07% of list prices set by delinquent homeowners are less than market value p.

$\mathcal{AWD}2$ is, mathematically, the same model as $\mathcal{AWD}$ and has the same parameter values, so it produces the same results. There is one critical difference. In $\mathcal{AWD}$, abovewater delinquent homeowners are prevented by search frictions from selling their homes to escape foreclosure, and so they lose their homes to foreclosure even though they have equity. In $\mathcal{AWD}2$ the same borrowers are underwater after accounting for the equity shock, which is why they do not sell.[59] All delinquent homeowners with positive effective equity who do not get current in $\mathcal{AWD}2$ will sell to avoid foreclosure. Therefore negative effective equity is a necessary condition for foreclosure in $\mathcal{AWD}2$, as it is in standard default models, and yet $\mathcal{AWD}2$ produces the same results as $\mathcal{AWD}$. This demonstrates that the critical mechanism in $\mathcal{AWD}$ is that homeowners who have equity when they become delinquent are still at substantial risk of foreclosure. A house price drop or other negative equity "trigger" independent of delinquency is not necessary to cause default, although delinquency can cause negative equity which in turn causes foreclosure.

Given its assumption of frequent and large equity shocks, two aspects of $\mathcal{AWD}$ that $\mathcal{AWD}2$ does not change are worth discussing. First, in $\mathcal{AWD}2$ equity shocks do not occur to delinquent borrowers who become current. This seems unrealistic, but if they did, they would lower the rate at which delinquent borrowers could escape foreclosure by becoming current. As shown in Table 5, this number is already too low in $\mathcal{AWD}$, so lowering it further would worsen the fit of the model to the data. It is possible that allowing equity shocks to also be positive (representing e.g. a loan modification or principal forgiveness) could address this issue, but there is no data to discipline this mechanism and so I do not pursue it further.

Second, in $\mathcal{AWD}2$ these negative equity shocks do not increase the lender's losses from foreclosure. Otherwise, they would reduce the number of third-party foreclosure sales and increase lenders' chargeoff rates, both of which would worsen the fit of the model to the data as shown in Table 5. This assumption is also necessary to preserve the results from Section 6.3 that, as in the data, recourse does not lower aggregate default rates, even though it lowers underwater default rates, because even under recourse lenders cannot sue defaulters who (in $\mathcal{AWD}$) have equity at the time of foreclosure.

[59] $\pi_s(p^l)$ in $\mathcal{AWD}2$ can even represent the probability that search frictions prevent a homeowner from selling, just as in $\mathcal{AWD}$, but with a different definition of the property's "value". In $\mathcal{AWD}$, property value is defined as the price p a non-distressed seller would get for the property. In $\mathcal{AWD}2$, property value can instead be defined as the price a seller can get for a property. Homeowners in foreclosure would accept any price at least equal to the value of outstanding mortgage debt, so any homeowner who cannot sell her home to escape foreclosure is by this definition underwater.

A.3 Value Functions

This subsection discusses the value functions that result from the model described in Section 3. All agents' welfare depends, in part, on their assets a and income y. As in many other consumption-savings models, solving the model is simpler if the agent's assets and her temporary income are combined into one variable called "cash-on-hand", denoted $\tilde{a}$. Income without the temporary component, $\overline{y}$, is given by Equation 2 and is useful for predicting future income.

Bequest Motives Let w denote an agent's wealth, which is the sum of her cash-on-hand $\tilde{a}$ and her effective home equity (if positive). Then her bequest value function is $B(w)$, where B is given by Equation 5. Mortality risk and bequest motives enter value functions in the standard rational, risk-neutral way, and they complicate the value functions without adding much insight. Therefore to simplify notation, for every continuation value function $V(\cdot)$, let $\hat{V}(\cdot)$ denote the continuation value function accounting for mortality risk; that is, $\hat{V}(\cdot) = (1-\pi_m)V(\cdot) + \pi_m B(w)$ where $\pi(m)$ is the age-dependent probability of mortality.

Non-homeowners Consider the problem of an agent who does not own a home. Denote her value function by V_t^N, and note that it is a function of assets a, default flag d, and income y.

The agent has two options. First, she can continue to rent a home, in which case she chooses nonhousing consumption c, rent r, and assets next period $a^{'}$, and her Bellman equation is:

$$V_t^{NR}(\tilde{a}, d, \overline{y}) = \max_{c,r,a^{'}} u(\frac{c}{e}, \frac{r}{e}) + \beta E_{\tilde{a}^{'},d^{'},\overline{y}^{'}} \hat{V}_{t+1}^N(\tilde{a}^{'}, d^{'}; \overline{y}^{'}) \tag{10}$$

subject to:

$$a' \geq \xi \overline{y} \tag{11}$$

$$R^{-1}a' + c + r = \tilde{a} + \overline{y} \tag{12}$$

where Equation 11 is the borrowing limit, and Equation 12 is the budget constraint.

An agent without a default flag ($d = 0$) can also buy a home of price p and obtain a mortgage of size L with payments m. L is determined endogenously by the no-profit condition for lenders, and is defined later. After purchasing a home, her psychic moving costs are drawn from the age-dependent distribution Ψ_t. Home purchases occur before consumption, and agents must pay their mortgage as scheduled the period they buy a home. Let V_t^{CP} (defined later) denote the value function of a homeowner who is current on her mortgage and pays it as scheduled. A nonhomeowner who buys a home immediately becomes a current homeowner and her value function is:

$$V_t^{NB}(\tilde{a}, d = 0; \overline{y}) = \max_{p,m} E_\psi V_t^{CP}(\tilde{a}_{new}, p, m; \psi, \overline{y}) \tag{13}$$

subject to:

$$\tilde{a}_{new} = \tilde{a} + \overline{y} + L(\tilde{a}, p, m, \overline{y}) - (1 + \phi_b)h \tag{14}$$

where Equation 14 gives the agent's new cash-on-hand, after receiving the mortgage and paying for the house.

The value function for an agent buying a home with a default flag is similar, except with the restriction that $m = 0$ which implies $L = 0$.

An agent who is not exogenously prohibited from buying a home but does not own one chooses to rent or to buy optimally, and so:

$$V_t^N(\tilde{a}, d; \overline{y}) = \max\{V_t^{NR}, V_t^{NB}\} \tag{15}$$

An agent who is exogenously prohibited from buying a home must rent one.

Current homeowners Now, consider the problem of an agent with a home who is current on her mortgage. Denote her value function by V_t^C, and note that it is a function of assets a, house h, mortgage payment p, income y and psychic moving costs ψ. The agent has four options: (1) pay her mortgage as scheduled, (2) refinance her mortgage, (3) sell the home, or (4) become delinquent on the mortgage.

If the agent pays her mortgage m as scheduled, she also pays property taxes $\zeta_t p$ and home maintenance costs $\zeta_m p$. The home provides housing services κp. Her Bellman equation is:

$$V_t^{CP}(\tilde{a}, p, m; \overline{y}, \psi) = \max_{c,a'} u(\frac{c}{e}, \frac{\kappa p}{e}) + \beta E_{\tilde{a}', p', \overline{y}', \psi'} \hat{V}_{t+1}^C(\tilde{a}', p', m; \overline{y}', \psi') \tag{16}$$

subject to:

$$a' \geq \xi \overline{y} \tag{17}$$

$$R^{-1}a' + c + m + (\zeta_t + \zeta_m)p = \tilde{a} + \overline{y} \tag{18}$$

where Equation 17 is the borrowing limit, and Equation 18 is the budget constraint.

The agent can also choose a new mortgage payment m_{new}. Mortgages do not have prepayment penalties, so if $m_{new} < m_{old}$, the agent does not pay any refinancing fees and her cash-on-hand is simply adjusted downwards by the change in her mortgage balance $\Pi(m_{old}, t) - \Pi(m_{new}, t)$. This cannot bring her cash-on-hand below the borrowing limit. The formula for Π is given by Equation 7.

However, if $m_{new} > m_{old}$, the agent must pay refinancing fees $\phi_R \Pi(m_{new}, t)$ to obtain a new mortgage. She receives a lump-sum amount $L(\tilde{a}, p, m_{new}, \overline{y})$, which she must use to pay off her old mortgage at cost $\Pi(m_{old}, t)$. After refinancing her mortgage, the agent must pay it as scheduled for the period. Cash-out refinances are also subject to the PTI and LTV constraints on new mortgages. Thus the Bellman equation for an agent taking a cash-out-refinance is:

$$V_t^{CR}(\tilde{a}_{old}, p, m_{old}; \overline{y}, \psi) = \max_{m_{new}} V_t^{CP}(\tilde{a}_{new}, p, m_{new}; \overline{y}, \psi) \tag{19}$$

subject to:

$$\tilde{a}_{new} = \tilde{a}_{old} + \tilde{L}(\tilde{a}, p, m_{new}, \overline{y}) - \Pi(m_{old}, t) - \mathbb{1}(m_{new} > m_{old})\phi_R \Pi(m_{new}, t) \tag{20}$$

$$\frac{m_{new} + \zeta_t p}{\overline{y}} \leq \text{PTI}^{\max} \tag{21}$$

$$100 * \frac{\Pi(m_{new}, t)}{p} \leq \text{LTV}^{\max} \tag{22}$$

where Equation 20 gives the agent's new cash-on-hand after refinancing the mortgage, Equation 21 is the PTI limit, and Equation 22 is the LTV limit. Note Equation 21 plays an important role in the model, by preventing many (though not all) abovewater homeowners in the "disaster" income state from taking out a cash-out refi to avoid defaulting.

The agent can also sell her home. After paying transaction costs, she receives $(1 - \phi_s)p$. She must use these proceeds to repay her mortgage debt. If she is effectively underwater, she can use her cash-on-hand to cover the difference between the mortgage debt and the house sale proceeds, but she cannot violate the borrowing constraint to do so. She pays psycholigical moving cost ψ to move out her home and becomes a nonhomeowner without a default flag. She can immediately buy another home if she chooses to. Thus the value function of a current agent selling her home is:

$$V_t^{CS}(\tilde{a}, p, m; \overline{y}, \psi) = (1 + \psi) \cdot V_t^N(\tilde{a} + (1 - \phi_s)p - \Pi(m, t), d = 0; \overline{y}) \tag{23}$$

Note that ψ in Equation 23 plays a fundamental role in this paper. It makes many effectively abovewater homeowners choose to become delinquent on their mortgages rather than sell their homes after adverse income shocks, unlike in other models.

Finally, a current homeowner can choose to become delinquent. If she does, her problem is similar to that of a current homeowner paying her mortgage, given by Equation 16 - Equation 18. There are two differences. First, she does not have to pay her mortgage, property taxes or maintenance fees. Second, at the beginning of the next period, she will not be current but instead will be delinquent and at risk of foreclosure. This means her continuation value function is given not by V^C but by V^D (defined later in Equation 29).

$$V_t^{CD}(\tilde{a}, p, m; \overline{y}, \psi) = \max_{c, a'} u(\frac{c}{e}, \frac{\kappa p}{e}) + \beta E_{\tilde{a}', p', \overline{y}', \psi'} \hat{V}_{t+1}^D(\tilde{a}', p', m; \overline{y}', \psi') \tag{24}$$

subject to:

$$a' \geq \xi \overline{y} \tag{25}$$

$$R^{-1}a' + c = \tilde{a} + \overline{y} \tag{26}$$

A current homeowner chooses whether to pay her mortgage, refinance her mortgage, sell her home, or become delinquent optimally, and so:

$$V_t^C(\tilde{a}, p, m; \overline{y}, \psi) = \max\{V_t^{CP}, V_t^{CR}, V_t^{CS}, V_t^{CD}\} \tag{27}$$

Delinquent homeowners A delinquent homeowner owes her lender what a current homeowner would owe $\Pi(m, t)$ plus her missed mortgage payment and back taxes with penalty interest $R_m^d(m + \zeta_t p)$. To get current, she must also repair the depreciated home at cost $R_\zeta(\zeta_m)p$. Recall that $\Phi_F(p, m, t) = R_m^d(m + \zeta_t p) + R_\zeta(\zeta_m)p$ denotes the costs the borrower must pay to become current.

At the beginning of the period, a delinquent homeowner can choose a list price p^l at which to try to sell her home. After a successful home sale, the agent must repay her mortgage debt including foreclosure fees, $\Pi(m,t) + \Phi_F(p,m,t)$. If she does, she becomes current without a foreclosure flag. Thus a delinquent homeowner who successfully sells her home at price p^l has value function:

$$V_t^{DS}(p^l; \tilde{a}, p, m; \overline{y}, \psi) = (1+\psi) \cdot V_t^N(\tilde{a} + (1-\phi_s)p^l - \Pi(m,t) - \Phi_F(p,m,t), d=0; \overline{y}) \tag{28}$$

A home listed for price p^l sells with probability $\pi_s(p^l)$. Recall the restriction that $\pi_s(p^l) = 0$ for $p^l > p$. Let $V_t^{D\not{S}}$ (defined later) denote the value function of a delinquent homeowner who does not sell her home. A delinquent homeowner chooses her list price optimally, and so:

$$V_t^D(\tilde{a}, p, m; \overline{y}, \psi) = \max_{p^l} (1 - \pi_s(p^l)) \cdot V_t^{D\not{S}}(\tilde{a}, p, m; \overline{y}, \psi) + \pi_s(p^l) \cdot V_t^{DS}(p^l; \tilde{a}, p, m; \overline{y}, \psi) \tag{29}$$

If the agent tries and fails to sell her home, or does not try to sell her home (equivalently, sets $p^l > p$), she may become current on her mortgage by paying $\Phi_F(p,m,t)$ out of her cash-on-hand. Recall that homeowners in the disaster income state are exogenously prohibited from getting current (see footnote 21), but that homeowners who do get current can immediately sell their home, refinance their mortgage, become delinquent again, or pay their mortgage on time. Thus the value function for a delinquent homeowner becoming current is:

$$V_t^{DC}(\tilde{a}, p, m; \overline{y}, \psi) = V_t^C(\tilde{a} - \Phi_F(p,m,t), p, m; \overline{y}, \psi) \tag{30}$$

A delinquent homeowner who does not sell her home and does not get current loses her home to foreclosure. The lender seizes the home and sells it for $(1-\chi_D)p$. If there are excess proceeds, they are returned to the agent. Otherwise, the agent keeps her cash-on-hand since there is no recourse under the baseline model. The agent pays psychic moving costs ψ and becomes a nonhomeowner with a default flag. Thus the value function for a delinquent homeowner who loses her home to foreclosure is:

$$V_t^{DF}(\tilde{a}, p, m; \overline{y}, \psi) = (1+\psi) V_t^N(\tilde{a} + \max\{0, (1-\chi_D)p - \Pi(m,t) - \Phi_F(p,m,t)\}, d = d^{max}; \overline{y}) \tag{31}$$

Finally, the value function $V_t^{D\not{S}}$ for a delinquent homeowner who does not sell is equal to V_t^{DF} if the agent has the disaster income state (since she is then prohibited from getting current), and is equal to $\max\{V_t^{DC}, V_t^{DF}\}$ otherwise.

Lenders Banks are risk-neutral and discount the future at rate $\beta_B = (R^m)^{-1}$. Let M_t denote the value to a bank of a mortgage owed by a current homeowner with state $(\tilde{a}, p, m; \overline{y}, \psi)$, and let M_t^D denote the value to a bank of a mortgage owed by a delinquent homeowner. When a bank extends a mortgage of size L to a home buyer, its return on the transaction is $M_t(\tilde{a}, p, m; \overline{y}, \psi) - L$. I assume that the mortgage market is perfectly competitive, which implies that at the time of origination L solves:

$$L = M_t(\tilde{a}, p, m; \overline{y}, \psi) \tag{32}$$

Let $D_t(\tilde{a}, p, m; \overline{y}, \psi)$ denote the decision rule of a current agent about her mortgage, with $D_t = 0$ for payment, $D_t = 1$ for making an extra payment, $D_t = 2$ for prepayment (either through selling or a cashout refinance), and $D_t = 3$ for delinquency. Then $M_t(\tilde{a}, p, m; \overline{y}, \psi)$ satisfies the recursion:

$$M_t(\tilde{a}, p, m; \overline{y}, \psi) = \begin{cases} m + \beta_b E(M_{t+1}(\tilde{a'}, \tilde{p'}, m; \overline{y}', \psi')) & \text{if } D_t = 0, \\ \Pi(m,t) - \Pi(m_{new}, t) + m_{new} + \beta_b E(\tilde{a'}, \tilde{p'}, m_{new}; \overline{y}', \psi')) & \text{if } D_t = 1, \\ \Pi(m,t) & \text{if } D_t = 2, \\ \beta_b E(M_{t+1}^D(\tilde{a'}, \tilde{p'}, m; \overline{y}', \psi')) & \text{if } D_t = 3, \end{cases} \tag{33}$$

Let $D_t^D(\tilde{a}, p, m; \overline{y}, \psi)$ denote the decision rule of a delinquent agent about her mortgage, with $D_t^D = 0$ for becoming current, $D_t^D = 1$ for selling, and $D_t^D = 2$ for foreclosure. Then $M_t^D(\tilde{a}, p, m; \overline{y}, \psi)$ satisfies the recursion:

$$M_t^D(\tilde{a}, p, m; \overline{y}, \psi) = \begin{cases} R^m m + M_t(\tilde{a}, p, m; \overline{y}, \psi) & \text{if } D_t = 0, \\ R^m m + \Pi(m,t) & \text{if } D_t = 1, \\ \min\{R^m m + \Pi(m,t), (1 - \chi_D)p\} & \text{if } D(a,h,y,m,p,t) = 2, \end{cases} \tag{34}$$

Note that, while delinquent homeowners pay a penalty interest rate R_m^d to get current, lenders receive only R^m. This is equivalent to assuming that any costs a lender incurs to start and then cancel foreclosure proceedings are precisely canceled out by the additional interest they charge delinquent borrowers. Thus lenders neither lose nor profit from canceled foreclosures.

Equations (32), (33) and (34) implicitly define the mortgage size new home buyers receive L.

Recourse Define $\underline{a}$ as the lowest possible cash-on-hand the agent can have. I assume that under recourse, if the value of the foreclosed property $p - R_\zeta(\zeta_m)p$ is less than the borrower's outstanding mortgage debt $\Pi(m,t)+R_m^d(m+\zeta_t p)$, the lender can costlessly seize a deficiency judgment DJ =max $\{0,\min\{\frac{\tilde{a}-\underline{a}}{2}, \Pi(m,t)+R_m^d(m+\zeta_t p)-(p-R_\zeta(\zeta_m)p)\}\}$. Under recourse, the value functions for all agents are the same as without recourse, except if an agent loses her home to foreclosure her assets are adjusted downards by DJ. That is:

$$V_t^{DF}(\tilde{a},p,m;\overline{y},\psi)=(1+\psi)V_t^N(\tilde{a}-DJ+\max\{0,(1-\chi_D)p-\Pi(m,t)-\Phi_F(p,m,t)\},d=d^{max};\overline{y}) \quad (35)$$

Lenders' proceeds in the event of forecosloure are adjusted upwards by the deficiency judgment DJ.

Forbearance A delinquent borrower who is offered forbearance can turn it down, allow the foreclosure to proceed, and obtain value function V_t^{DF} given by Equation 31. If she accepts forbearance, she must make up her missed mortgage payment m from the previous period, but she is not charged any interest or fees. She also must make her current mortgage payment as scheduled. She cannot refinance her mortgage for the period, but she can sell her home to make these payments.

If she stays in her home, her Bellman equation is very similar to that of a current homeowner paying (Equation 16) except she makes two mortgage payments. Thus the value function for a homeowner in forbearance paying is V_t^{FP} where:

$$V_t^{FP}(\tilde{a},p,m;\overline{y},\psi)=\max_{c,a'} u(\frac{c}{e},\frac{\kappa p}{e})+\beta E_{\tilde{a}',p',\overline{y}',\psi'}\hat{V}_{t+1}^C(\tilde{a}',p',m;\overline{y}',\psi') \quad (36)$$

subject to:

$$a'\geq \xi\overline{y} \quad (37)$$

$$R^{-1}a'+c+2m+(\zeta_t+\zeta_m)p=\tilde{a}+\overline{y} \quad (38)$$

If she sells her home, her value function is very similar to that of a current homeowner selling (Equation 23) except she must make up her missed payment. Thus the value function for a homeowner in forbearance selling is V_t^{FC} where:

$$V_t^{FS}(\tilde{a},p,m;\overline{y},\psi)=(1+\psi)\cdot V_t^N(\tilde{a}+(1-\phi_s)p-\Pi(m,t)-m,d=0;\overline{y}) \quad (39)$$